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(1)

C3 June 06 – Solutions

1. a) $\frac{3x^2-x-2}{x^2-1} = \frac{(3x+2)(x-1)}{(x-1)(x+1)} = \frac{3x+2}{x+1}$

b) $\frac{3x^2-x-2}{(x^2-1)} - \frac{1}{x(x+1)} = \frac{3x+2}{x+1} - \frac{1}{x(x+1)}$

$= \frac{3x^2+2x-1}{x(x+1)}$

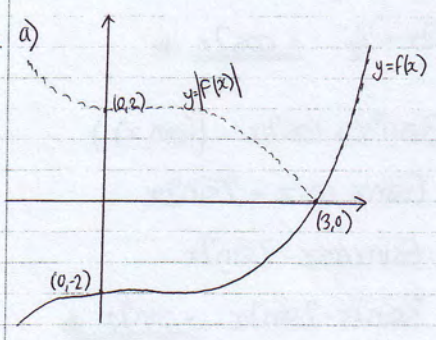
$= \frac{(3x-1)(x+1)}{x(x+1)} = \frac{3x-1}{x}$

2. a) $y = e^{3x} + \ln(2x)$ $\frac{dy}{dx} = 3e^{3x} + \frac{2}{2x} = 3e^{3x} + \frac{1}{x}$

b) $y = (5+x^2)^{3/2}$ $\frac{dy}{dx} = \frac{3}{2}(5+x^2)^{1/2} \times 2x$

$= 3x(5+x^2)^{1/2}$

3. a)



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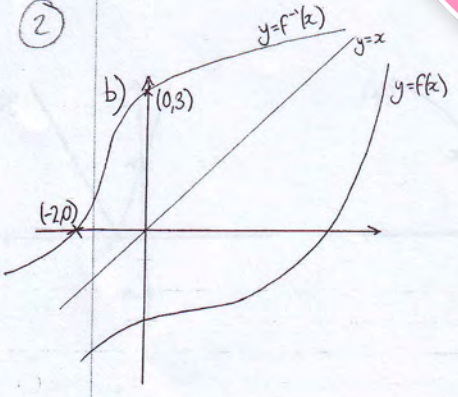
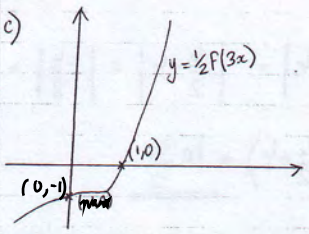
b)



c)



4. a) when $t=0$ $T = 400+25 = 425°C$

b) $300 = 400e^{-0.05t} + 25$

$\frac{275}{400} = e^{-0.05t}$

$-0.05t = \ln\left(\frac{275}{400}\right)$ $t = \frac{1}{-0.05}\ln\left(\frac{275}{400}\right) = 7.49 \text{ min}$ (3)

c) Rate of change $\frac{dT}{dt} = -0.05 \times 400e^{-0.05t}$

$= -20e^{-0.05t}$

(3)

$\left.\frac{dT}{dt}\right|_{t=50} = -20e^{-0.05\times 50} = -1.64 °C \text{ per min}$ (3sf).

d) The min value of $400e^{-0.05t}$ is >0 $\therefore$ T is always >25.

(4)

5. a) $y = (2x-1)\tan 2x$ $u = (2x-1)$ $u' = 2$ $v = \tan 2x$ $v' = 2\sec^2 2x$

$\frac{dy}{dx} = 2(2x-1)\sec^2 2x + 2\tan 2x$

minimum point when $\frac{dy}{dx} = 0$

$\therefore (4x-2)\sec^2 2x + 2\tan 2x = 0$

$\frac{(4x-2)}{\cos^2 2x} + \frac{2\sin 2x}{\cos 2x} = 0$

$\frac{4x-2}{\cos^2 2x} + \frac{2\sin 2x\cos 2x}{\cos^2 2x} = 0$

$\therefore 4x-2 + 2\sin 2x\cos 2x = 0$

$4x-2 + \sin 4x = 0$

at P $x=k$ $\therefore$ $4k-2+\sin 4k = 0$

(4)

6. a) $\sin^2\theta + \cos^2\theta \equiv 1$ $\div \sin^2\theta$

$1 + \cot^2\theta \equiv \operatorname{cosec}^2\theta$

$\cot^2\theta - \operatorname{cosec}^2\theta \equiv -1$

$\operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ #

b) LHS $= \operatorname{cosec}^4\theta - \cot^4\theta = (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^2\theta +$

$= \operatorname{cosec}^2\theta + \cot^2\theta$ #

c) $\operatorname{cosec}^2\theta + \cot^2\theta = 2 - \cot\theta$ (from b)).

$1 + \cot^2\theta + \cot^2\theta = 2 - \cot\theta$ (from a))

$2\cot^2\theta + \cot\theta - 1 = 0$

$(2\cot\theta - 1)(\cot\theta + 1) = 0$

$\cot\theta = \frac{1}{2}$ $\cot\theta = -1$

$\tan\theta = 2$ $\tan\theta = -1$

$\theta = 63.4°$ $\theta = -45°, 135°, \ldots$

$\theta = 135°$ since $90° < \theta < 180°$

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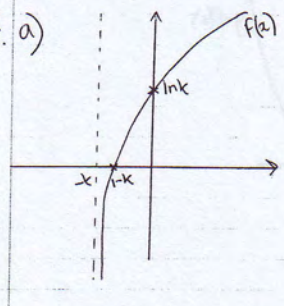
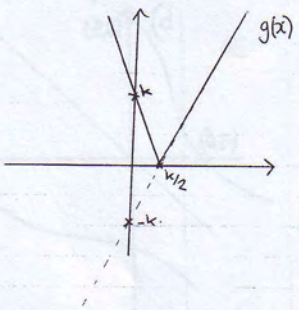
(5)

7. a)





b) $x \in \mathbb{R}$.

c) $g\left(\frac{k}{4}\right) = \left|\frac{2k}{4} - k\right| = \left|\frac{k}{2} - k\right| = \left|-\frac{k}{2}\right| = \frac{k}{2}$

$fg\left(\frac{k}{4}\right) = \ln\left(\frac{k}{2} + k\right) = \ln\frac{3k}{2}$

d) $9y = 2x + 1$

$y = \frac{2}{9}x + \frac{1}{9}$ Grad of tangent $= \frac{2}{9}$

$y = \ln(x + k)$

$\frac{dy}{dx} = \frac{1}{x+k}$

$\left.\frac{dy}{dx}\right|_{x=3} = \frac{1}{3+k} = \frac{2}{9}$

$9 = 2(3+k)$ $\quad k = 1.5$

(6)

8 a) $\cos A = \frac{3}{4}$

$\cos^2 A + \sin^2 A = 1$

$\frac{9}{16} + \sin^2 A = 1$

$\sin A = \pm\sqrt{1 - \frac{9}{16}} = \pm\frac{\sqrt{7}}{4}$

$\therefore \sin 2A = 2\sin A \cos A$

$\sin 2A = 2 \times \pm\frac{\sqrt{7}}{4} \times \frac{3}{4} = \pm\frac{6\sqrt{7}}{16} = \pm\frac{3\sqrt{7}}{8}$

b) i) $\cos\left(2x + \frac{\pi}{3}\right) + \cos\left(2x - \frac{\pi}{3}\right)$

$= \cos 2x \cos\pi/3 - \sin 2x \sin\pi/3 + \cos 2x \cos\pi/3 + \sin 2x \sin\pi/3$

$= 2\cos 2x \cos\pi/3$

$= 2\cos 2x \times \frac{1}{2} = \cos 2x$ #

ii) $y = 3\sin^2 x + \cos 2x$ (from i))

$\frac{dy}{dx} = 6\sin x \cdot \cos x - 2\sin 2x$

$= 6\sin x \cos x - 2\sin 2x$

$= 3\sin 2x - 2\sin 2x = \sin 2x$ #