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C3 Jan07 - Solutions

1 a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$

$= 2\sin\theta\cos\theta\cos\theta + (\cos^2\theta - \sin^2\theta)\sin\theta$

$= 2\sin\theta\cos^2\theta + \sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta$

$= 3\sin\theta(1-\sin^2\theta) - \sin^3\theta$

$= 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$= \underline{3\sin\theta - 4\sin^3\theta}$ #

b) $\sin\theta = \frac{\sqrt{3}}{4} \Rightarrow \sin 3\theta = \frac{3\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3$

$= \frac{3\sqrt{3}}{4} - 4\times\frac{3\sqrt{3}}{64}$

$= \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \underline{\frac{9\sqrt{3}}{16}}$

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2 a) $f(x) = 1 - \frac{3}{(x+2)} + \frac{3}{(x+2)^2}$

$= \frac{(x+2)^2 - 3(x+2) + 3}{(x+2)^2}$

$= \frac{x^2+4x+4-3x-6+3}{(x+2)^2}$

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$= \underline{\frac{x^2+x+1}{(x+2)^2}}$ #

b) $x^2+x+1 = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} > 0$ #

c) Numerator >0 from b)

Denominator $>0 \Rightarrow f(x) > 0$. #

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3 a) $x = 2\sin y$

when $y = \pi/4$ $x = 2\sin\pi/4 = 2\times\frac{\sqrt{2}}{2} = \underline{\sqrt{2}}$ #

b) $\frac{dx}{dy} = 2\cos y \Rightarrow \frac{dy}{dx} = \frac{1}{2\cos y}$

when $y = \pi/4$ $\frac{dy}{dx} = \frac{1}{\cos\pi/4} = \frac{1}{2\times\sqrt{2}/2} = \underline{\frac{1}{\sqrt{2}}}$ #

c) Grad of tangent at P $= \frac{1}{\sqrt{2}}$

$\therefore$ Grad of normal $= -\sqrt{2}$

$\therefore$ eqn. of normal $y - \pi/4 = -\sqrt{2}(x-\sqrt{2})$

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$y - \pi/4 = -\sqrt{2}x + 2$

$\underline{y = -\sqrt{2}x + 2 + \pi/4}$

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4. I) $y = x(9+x^2)^{-1}$

turning points when $\frac{dy}{dx} = 0$.

$\frac{dy}{dx} = (9+x^2)^{-1} + x\times -(9+x^2)^{-2}\times 2x$

$= (9+x^2)^{-1} - 2x^2(9+x^2)^{-2}$

$\therefore \frac{1}{9+x^2} - \frac{2x^2}{(9+x^2)^2} = 0$

$\frac{9+x^2-2x^2}{(9+x^2)^2} = 0$

$9 - x^2 = 0$

$x = \pm 3$

when $x = 3$ $y = \frac{3}{9+(3)^2} = \frac{3}{18} = \frac{1}{6}$

$\therefore$ Coords $\underline{(3, 1/6)}$ and $\underline{(-3, -1/6)}$

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II) $y = (1+e^{2x})^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(1+e^{2x})^{1/2}\times 2e^{2x}$

$\frac{dy}{dx} = 3e^{2x}(1+e^{2x})^{1/2}$

$\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}\ln 3} = 3e^{\ln 3}(1+e^{\ln 3})^{1/2} = 9(1+3)^{1/2}$

$= \underline{18}$

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5 a) $y = \sqrt{3}\cos x + \sin x$

$R\sin(x+\alpha) = R\sin x\cos\alpha + R\sin\alpha\cos x$

$\therefore R\cos\alpha = 1 \qquad R\sin\alpha = \sqrt{3}$

$\tan\alpha = \sqrt{3}$

$\alpha = \pi/3$

$\therefore R = \frac{1}{\cos\pi/3} = 2$

$\therefore \underline{y = 2\sin(x+\pi/3)}$

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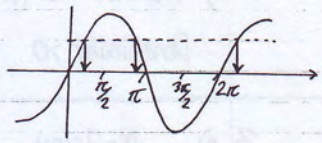
b) $1 = 2\sin(x+\pi/3)$ $\quad 0 \leq x \leq 2\pi$

$\sin(x+\pi/3) = \frac{1}{2}$

$x+\pi/3 = \sin^{-1}\frac{1}{2}$

$x+\pi/3 = \pi/6$ $\quad \pi/3 \leq x+\pi/3 \leq 7\pi/3$



$x+\pi/3 = \pi/6, 5\pi/6, 13\pi/6$

$x = -\pi/6, 3\pi/6, 11\pi/6$

$x = \pi/2, 11\pi/6$

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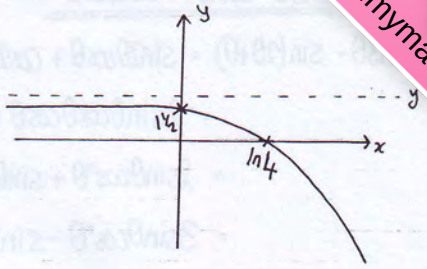
c)



meet x axis when $y=0$ $\quad 2-\frac{1}{2}e^x = 0$

$e^x = 4$

$x = \ln 4$

d) $x_1 = -\frac{1}{2}e^{-0.3} = -0.3704$ (4dp)

$x_2 = -\frac{1}{2}e^{-0.370409} = -0.3452$ (4dp)

e) $x_3 = -0.3540$
$x_4 = -0.3509$
$x_5 = -0.3520$
$x_6 = -0.3516$
$x_7 = -0.3517$ $\quad \therefore$ $x = -0.352$ (3dp).

6. a) $y = \ln(4-2x)$

$e^y = 4-2x$

$2x = 4-e^y$

$x = 2-\frac{1}{2}e^y$ $\quad \therefore F^{-1}: x \longmapsto 2-\frac{1}{2}e^x$ $\quad x \in \mathbb{R}$.

b) $x < 2$.

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7. a) $F(2) = (2)^4 - 4\times 2 - 8 = 16$

$F(1) = (1)^4 - 4\times 1 - 8 = -3$

$\therefore$ root exists between 1 and 2. #

b) turning point when $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 4x^3 - 4 = 0$

$x^3 - 1 = 0$

$x^3 = 1 \Rightarrow x = 1 \Rightarrow y = 1^4 - 4\times 1 - 8 = -11$

Coords $(1, -11)$

c)

$$x-2 \;\overline{)\; x^4 \qquad -4x-8}$$

x^3+2x^2+4x+4

$x^4 - 2x^3$

$2x^3 - 4x - 8$

$2x^3 - 4x^2$

$4x^2 - 4x - 8$

$4x^2 - 8x$

$4x - 8$

$4x - 8$

$\therefore$ $a=2, b=4, c=4$

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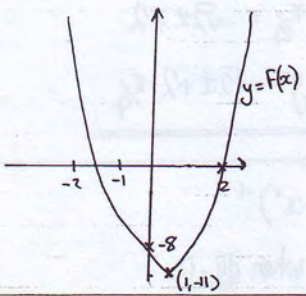
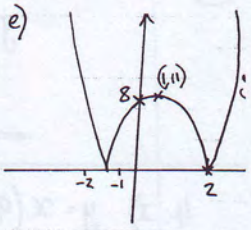
d)



e)



8 i) $\sec^2 x - \operatorname{cosec}^2 x \equiv \tan^2 x - \cot^2 x$

LHS $= \sec^2 x - \operatorname{cosec}^2 x$

$= \tan^2 x + 1 - (\cot^2 x + 1)$

$= \tan^2 x + 1 - \cot^2 x - 1 = \tan^2 x - \cot^2 x$ #

ii) $y = \arccos x$

a) $x = \cos y$

$x = \sin(\pi/2 - y)$

$\pi/2 - y = \arcsin x$

b) $\arccos x + \arcsin x$

$= y + \pi/2 - y$

$= \pi/2$