AS Further Mathematics 8FM0

Specimen Paper - Further Statistics 2 Mark Scheme

<table>
<tr><th>Qu</th><th>Scheme</th><th>Marks</th><th></th><th>AO</th></tr>
<tr><td>1</td><td>
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<tr><td>Plate</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td></tr>
<tr><td>Judge</td><td>7</td><td>3</td><td>2</td><td>1</td><td>8</td><td>10</td><td>4</td><td>6</td><td>5</td><td>9</td></tr>
<tr><td>Age</td><td>8</td><td>1</td><td>2</td><td>3</td><td>10</td><td>7</td><td>9</td><td>6</td><td>4</td><td>5</td></tr>
</table>
</td><td>M1
A1</td><td></td><td>1.1b
1.1b</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td>$\sum d^2 = 1+4+0+4+4+9+25+0+1+16 \quad [=64]$</td><td>M1</td><td></td><td>1.1b</td></tr>
<tr><td></td><td>$r_s = 1-\frac{6\times"64"}{10\times99}$</td><td>M1</td><td></td><td>1.1b</td></tr>
<tr><td></td><td>$= 0.61212\ldots$ = awrt <u>0.612</u></td><td>A1</td><td></td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(5 marks)</td><td></td><td></td></tr>
<tr><td></td><td>Notes</td><td colspan="3"></td></tr>
<tr><td></td><td>1st M1 an attempt to rank judge against actual ages</td><td colspan="3"></td></tr>
<tr><td></td><td>1st A1 both correct ranks</td><td colspan="3"></td></tr>
<tr><td></td><td>2nd M1 for an attempt to find $\sum d^2$ (some correct d values found and sum attempted)</td><td colspan="3"></td></tr>
<tr><td></td><td>3rd M1 for using their $\sum d^2$ in formula for r_s with $n = 10$</td><td colspan="3"></td></tr>
<tr><td></td><td>2nd A1 for awrt 0.612 or exact fraction e.g. $\frac{101}{165}$</td><td colspan="3"></td></tr>
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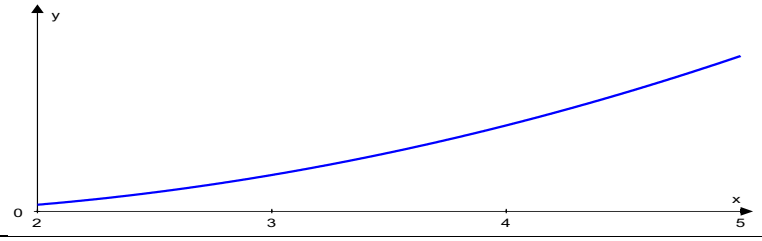
Qu	Scheme	Marks		AO
2(a)	e.g. $F'_C(x) = \frac{1}{6}(9-2x)$	M1		3.4
	For a cdf $F'(x)$ must be Ö 0 for all values of x but < 0	A1		2.4
		(2)		
(b)	$k\int\left(x^2-2x+\frac{2}{3}\right)\,dx = 1 \Rightarrow k\left[\frac{1}{3}x^3-x^2+\frac{2}{3}x\right]_{(2)}^{(5)} = 1$	M1		1.1b
	So $k\left[\left(\frac{5^3}{3}-5^2+\frac{2}{3}\times 5\right)-\left(\frac{2^3}{3}-2^2+\frac{2}{3}\times 2\right)\right]=1$	M1		1.1b
	$k = \frac{1}{20}$	A1		1.1b
		(3)		
(c)		B1 B1		1.1b 1.1b
		(2)		
(d)	$E(X) = \int_2^5 x f_D(x)\,dx = \frac{1}{20}\left[\frac{x^4}{4}-\frac{2}{3}x^3+\frac{1}{3}x^2\right]_{(2)}^{(5)}$	M1		1.1b
	$= \frac{1}{20}\left[\left(\frac{5^4}{4}-\frac{2}{3}\times 5^3+\frac{5^2}{3}\right)-\left(\frac{2^4}{4}-\frac{2}{3}\times 2^3+\frac{2^2}{3}\right)\right]$	M1		1.1b
	$= \underline{\mathbf{4.0625}}$	A1		1.1b
		(3)		
(e)	Sketch shows negative skew (with mode at 5)	B1ft		2.4
	Therefore expect that **mean < median**	dB1ft		2.2b
		(2)		
		(12 marks)		

	Notes
(a)	M1 for attempting to differentiate the cdf or a suitable sketch showing a max in the range
	A1 for a suitable explanation that shows the model is not appropriate.
(b)	1st M1 for attempting to integrate with some correct integration ($x^n \rightarrow x^{n+1}$) and set equal to 1
	2nd M1 for some correct use of limits of 2 and 5
	A1 for $k = \frac{1}{20}$
(c)	1st B1 for a curve (may look like a straight line) of positive gradient with f(2)> 0 and f(5) > f(2)
	2nd B1 for clear indication of 2 and 5 on x axis and f(x) = 0 elsewhere. [f(2)> 0 and f(5) > f(2)]

(d)	1st M1 for attempting $\int xf_D(x)\,dx$ and some correct integration ($x^n \rightarrow x^{n+1}$)
	2nd M1 for some correct use of limits of 2 and 5
	A1 for 4.0625 (accept 4.06 or 4.062 or 4.063)
(e)	1st B1ft for a suitable reason based on skewness or correct calculation of median = 4.21(44..)
	2nd dB1ft for stating that mean < median **with** some reference to part (c)
	NB A correct calculation of the median (sight of 4.21 or better) with no reference to the sketch will only score B1B0

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Qu	Scheme	Marks		AO
3	Area $= X(7 - X)$	M1		3.4
	$E(\text{Area}) = E(7X) - E(X^2)$ or $7\int_0^7 \frac{1}{7}x\,dx - \int_0^7 \frac{1}{7}x^2\,dx$	M1		2.1
	$E(X) = 3.5$ (from formula booklet or from integration)	B1		1.1b
	$E(X^2) = \text{Var}(X) + [E(X)]^2$ or $\int_0^7 \frac{1}{7}x^2\,dx = \left[\frac{x^3}{21}\right]_0^7$ $= \frac{49}{12} + [E(X)]^2$ [= 16.3333…]	M1		1.1b
	$E(\text{Area}) = 7\times"3.5" - \left[4.08\dot{3} + "3.5"^2\right]$	A1ft		2.1
	= 8.1666.. awrt **8.17**	A1		1.1b
		(6 marks)		

	Notes
	1st M1 for using the model to form expression for the area
	2nd M1 for using E(area) and splitting the integral
	B1 for $E(X) = 3.5$
	3rd M1 for use of a method for finding $E(X^2)$: formula with $\text{Var}(X)$ and $E(X)$ or integration
	1st A1 for a correct numerical expression for E(area), ft their $E(X)$
	2nd A1 for a final answer of awrt 8.17
	[Designated as a problem/extended solution]

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Qu	Scheme	Marks		AO
4(a)	$H_0 : \rho = 0 \quad H_1 : \rho < 0$	B1		2.5
	5% one-tail cv for r is: (–) 0.5494	M1		1.1b
	Significant result so there is evidence of a (-ve) correlation	A1		2.2b
		(3)		
(b)	Significant result suggests there is evidence of a linear relationship. So it is consistent.	B1		2.4
		(1)		
(c)	Need to choose model of the form: $h = a + bw$	M1		3.3
	Need S_{hw} so use $(S_{hw})^2 = r^2 \times S_{hh} \times S_{ww}$	M1		2.1
	Since $r < 0$ need negative square root so $S_{hw} = -189.4749...$	A1		2.3
	$b = \frac{S_{hw}}{S_{ww}} = \frac{"-189.4749..."}{470.5}$ [= – 0.402709…]	M1		3.4
	$a = 86.5 - "-0.402709..." \times 10.5$ [90.72845…]	M1		1.1b
	So $\boldsymbol{h = 90.7 - 0.403w}$	A1		1.1b
		(6)		
(d)	5"b" = 2.01…i.e. **loss** of approximately **2%**	B1		3.4
		(1)		
(e)	RSS = $S_{hh} \times (1 - r^2)$, = 122.1916… awrt **122**	M1, A1		1.1b (x2)
		(2)		
(f)	Eric's residual = $80 - "90.7 - 0.403" \times 10$ =, – 6.67 awrt **– 6.7**	M1, A1		3.4,1.1b
		(2)		
(g)(i)	Current model doesn't take account of hearing function before	B1		3.5b
	No "control group" to establish how hearing function changes for others not working in the factory			
(ii)	Instead of using variable h use a variable that measures difference in hearing function since working in factory	B1		2.4
		(2)		
		(17 marks)		

	Notes
(a)	B1 for both hypotheses correct in terms of ρ
	M1 for use of tables to find the cv of (condone +)
	A1 for a correct conclusion based on comparing – 0.620 with – 0.5494
(b)	B1 for mention of the result being significant and saying it is consistent
(c)	1st M1 for selecting the appropriate model
	2nd M1 for a suitable strategy/formula to find S_{hw}
	1st A1 for S_{hw} = awrt – 189
	3rd M1 for a correct expression for b (ft their S_{hw})
	4th M1 for a correct expression for a (ft their b)

	2nd A1 for a correct equation with a = awrt 90.7 and b = awrt – 0.403
(e)	M1 for use of a correct formula (some correct substitution seen)
	A1 for awrt 122
(f)	M1 for a correct calculation using their line from (c) (allow $\pm$)
	A1 for awrt – 6.7
(g)(i)	B1 for a suitable comment illustrating limitation of the current model
(ii)	B1 for suggesting change of variable h to use idea of difference in hearing function