# AS Further Mathematics 8FM0

## Specimen Paper – Decision Mathematics 2 Mark Scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1(a) | (i) The incomplete data must be dealt with by creating entries (significantly) larger than any other entries in the table (e.g. at least twice the largest entry) | B1 | 3.5c |
| | (ii) This allows these cells to be sufficiently "unattractive" that they will not be allocated. | B1 | 2.4 |
| | | **(2)** | |

**(b)**

| | P | Q | R | S |
|---|---|---|---|---|
| A | 7 | 8 | 18 | 6 |
| B | 8 | 21 | **42** | 12 |
| C | 6 | 17 | 9 | **42** |
| D | 16 | 20 | 13 | 11 |

Reducing rows and then columns — M1 (2.1)

| | P | Q | R | S |
|---|---|---|---|---|
| A | 1 | 2 | 12 | 0 |
| B | 0 | 13 | **34** | 4 |
| C | 0 | 11 | 3 | **36** |
| D | 5 | 9 | 2 | 0 |

| | P | Q | R | S |
|---|---|---|---|---|
| A | 1 | 0 | 10 | 0 |
| B | 0 | 11 | **32** | 4 |
| C | 0 | 9 | 1 | **36** |
| D | 5 | 7 | 0 | 0 |

A1 (1.1b)

Fewer than 4 lines required to cover the zeros hence solution is not optimal – augment by 1 — B1 (2.4)

| | P | Q | R | S |
|---|---|---|---|---|
| A | 2 | 0 | 10 | 0 |
| B | 0 | 10 | **31** | 3 |
| C | 0 | 8 | 0 | **35** |
| D | 6 | 7 | 0 | 0 |

M1 (2.1)
A1 (1.1b)

Four lines now required to cover the zeros hence solution is now optimal.

A-Q, B-P, C-R, D-S, minimum cost is 36. — A1ft (2.2a)

**(6)**

**(8 marks)**

# AS Further Mathematics 8FM0

## Specimen Paper – Decision Mathematics 2 Mark Scheme

**Question 1 notes:**

ai1B1 Reference as to how to refine model – not able to deal with incomplete data – indication that blank cells must be given a large value

aii1B1 Explanation of why this adaptation works –cell values are too large to be included in a minimising solution

b1M1 blank cells filled in with a number $\geq 42$ (condone $> 21$) and an attempt to reduce rows and then columns

b1A1 CAO

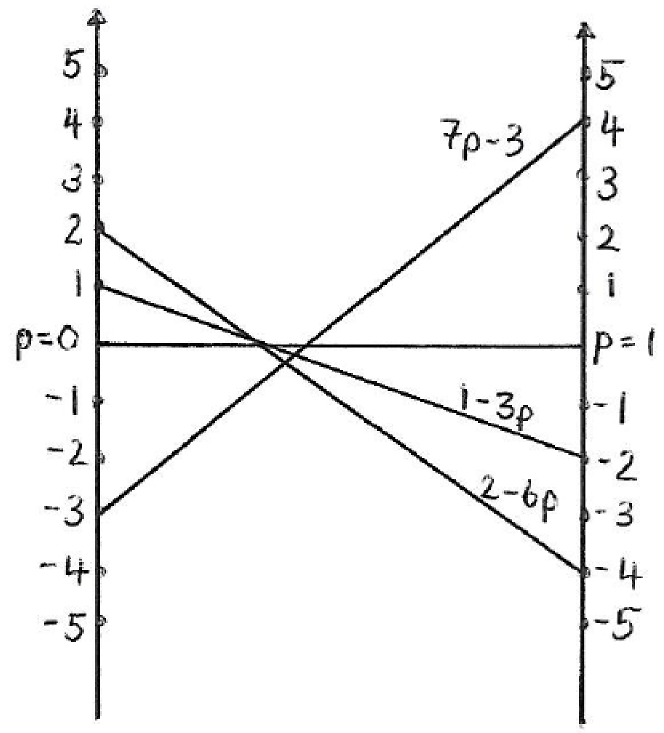
b2B1 At least one correct statement regarding both min number of lines to cover zeros and augmentation (for this or final table)

b2M1 Develop an improved solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 3 lines needed to 4 lines needed (so getting to the optimal table)

b2A1 correct final table

b3A1ft deduction of the correct allocation and minimum cost from their final table, and both statements for optimality

## AS Further Mathematics 8FM0
## Specimen Paper – Decision Mathematics 2 Mark Scheme

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | If a player looks for the worst that could happen if he makes each choice in turn, then… | B1 | 2.4 |
| | …his "play safe" strategy is the choice that results in the least worst option for him. | B1 | 1.2 |
| | | **(2)** | |
| **(b)** | Row minima: −4, −2, −1 max is −1<br>Column maxima: 4, 3 min is 3 | M1 | 1.1b |
| | Row maximin (−1) ≠ Col minimax (3) so not stable | A1 | 2.2a |
| | | **(2)** | |
| **(c)** | Let $B$ play 1 with probability $p$ and let $B$ play 2 with probability $1-p$, and using this to get at least one equation in $p$ | B1 | 3.3 |
| | Then if $A$ plays 1, B's gains are $-[-4p+3(1-p)] = 7p-3$<br>If $A$ plays 2, B's gains are $-[4p-2(1-p)] = 2-6p$<br>If $A$ plays 3, B's gains are $-[2p-1(1-p)] = 1-3p$<br><br>(See Notes: $3-7p$, $6p-2$, $3p-1$ may be correct alternatives) | M1<br>A1 | 1.1b<br>1.1b |
| |  | M1<br>A1 | 1.1b<br>1.1b |
| | Intersection of $7p-3$ and $2-6p$ occurs (where $p=\frac{5}{13}$) | dM1 | 1.1b |
| | Therefore player $B$ should play 1 with probability $\frac{5}{13}$ and 2 with probability $\frac{8}{13}$ | A1ft | 3.2a |
| | The value of the game to player $A$ is $\frac{4}{13}$ | A1 | 3.2a |
| | | **(8)** | |
| | | | **(12 marks)** |

**Question 2 notes**

a1B1 reference to considering all of the options for the "**worst**" that could happen each time

a2B1 defining "play safe" as selecting the "**least worst**" option

b1M1 Finding row minimums and column maximums – condone one error

b2A1 explanation involving $-1 \neq 3$ and a conclusion
NOTE there are numerous alternative ways to approach c), which can all gain full marks if they are consistent throughout. A candidate may omit the initial –, or they may transpose their matrix in terms of $B$ initially.

c1B1 Translating situation into model by defining variables

c1M1 constructing three probability expressions (need not be simplified)

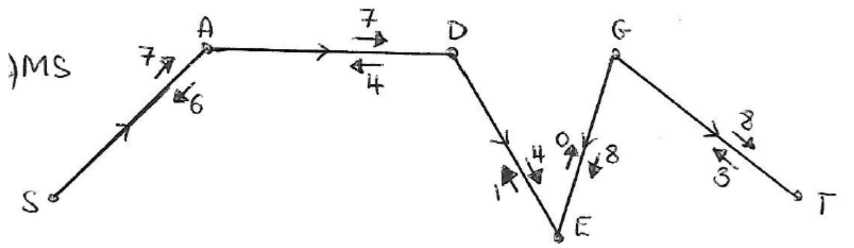
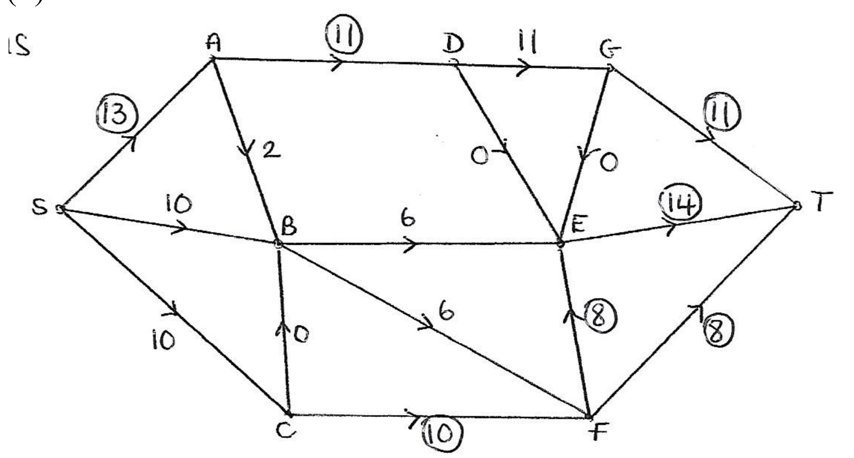
c1A1 all three probability expressions correct and simplified to form $ap + b$

c2M1 axes correct, at least one line correctly drawn for their expression

c2A1 correct graph
c3dM1 using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$

c3A1ft interpret their value of $p$ in the context of the question – must refer to play, player $B$

c4A1 cao for player $A$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | $K_1$ is not a cut as the source S and sink T are both on the same side of $K_1$. | B1 | 2.4 |
| | | **(1)** | |
| **(b)** | (i) $C_1 = 13 + 11 + 2 + 10 = 36$<br>$C_2 = 13 + 14 + 8 = 35$ | M1<br>A1 | 1.1b<br>1.1b |
| | (ii) Initial flow = 6 + 9 + 7 = 3 + 14 + 5 = 22 | B1 | 1.1b |
| | | **(3)** | |
| **(c)** |  | M1<br>A1 | 1.1b<br>1.1b |
| | | **(2)** | |
| **(d)** | (i)<br>e.g. SADGT = 7<br>SCFT = 3<br>SBEDGT = 1 | M1<br>A1<br>A1 | 1.1b<br>1.1b<br>1.1b |
| | (ii)<br> | B1 | 1.1b |
| | | **(4)** | |
| **(e)** | (i) Max flow = 22 + 11 = 33 | B1 | 1.1b |
| | (ii) e.g. a cut through the saturated arcs GT, ET & FT (has value 33). | M1 | 3.1a |
| | Therefore since max flow = min cut by max flow-min cut theorem, flow is maximal. | A1 | 2.1 |
| | | **(3)** | |
| | | | **(13 marks)** |

**Question 3 notes**

a1B1 explanation that S and T need to be on different sides for a cut

b1M1 indication of adding capacities, can be gained for either 36 or 35 seen

b1A1 cao

b1B1 cao

c1M1 any two arrows correct on one arc

c1A1 cao

d1M1 one flow augmenting route found from S to T
d1A1 two correct flow augmenting routes and flows

d2A1 all correct flow augmenting routes and flows, flow increased by 11

d1B1 a consistent flow pattern >26, one number on each arc

e1B1 max flow = 33

e2M1 selecting process of finding a cut to solve problem

e3A1 correct cut + value, constructing argument using max-flow min-cut theorem

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | $x_n = 1.03x_{n-1} - 600$<br>Auxiliary equation $\lambda - 1.03 = 0$ so $\lambda = 1.03$<br>Complementary function $A \times 1.03^n$ | B1 | 1.1b |
| | Try $x_n = \emptyset$ so $x_{n+1} = \emptyset$<br>$\emptyset = 1.03\emptyset - 600 \Rightarrow \emptyset = 20\,000$ | M1 | 1.1b |
| | $x_n = A \times 1.03^n + 20000$ | A1 | 1.1b |
| | Valid alternative method for (a):<br>$x_1 = 1.03x_0 - 600$<br>$x_2 = 1.03(1.03x_0 - 600) - 600$<br>$(x_2 = 1.03^2x_0 - 600(1 + 1.03))$<br>$x_3 = 1.03^3x_0 - 600(1 + 1.03 + 1.03^2)$<br>$x_n = 1.03^nx_0 - 600\left(\frac{1.03^n - 1}{1.03 - 1}\right)$ | | |
| | | **(3)** | |
| **(b)** | $x_n = -16000 \times 1.03^n + 20000$<br>or<br>$x_n = 4000 \times 1.03^n - 600\left(\frac{1.03^n - 1}{1.03 - 1}\right)$ | B1ft | 1.1b |
| | | **(1)** | |
| **(c)** | $600 > -16000 \times 1.03^n + 20000$ | M1 | 3.1b |
| | $\frac{97}{80} < 1.03^n$ | A1 | 1.1b |
| | Using logs to give $6.52 < n$<br>Hence when $n = 7$<br>or the end of the 7th year | A1 | 3.2a |
| | | **(3)** | |
| | | | **(7 marks)** |

**Question 4 notes:**

a1B1 for the correct complementary function

a1M1 correct method for finding a value for Ø

a1A1 cao (note alternative acceptable form)

b1B1ft substituting 4000 in to their general solution

c1M1 creating an equation or inequality involving 600 and their particular solution

c1A1 rearranging to the given simplified expression (oe)

c2A1 cao