Pearson Edexcel Level 3 GCE

Sample Assessment Material

Time: 1 hour 30 minutes | Paper Reference **9FM0/4D**

Further Mathematics
Advanced
Paper 4D: Decision Mathematics 2

You must have:
Decision Mathematics Answer Book (enclosed), calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read **each** question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

S61303A

1/1/1/1/

S 6 1 3 0 3 A

Pearson

Answer ALL questions. Write your answers in the answer book provided.

1. Table 1 shows the cost, in pounds, of transporting one unit of stock from each of four supply points, A, B, C and D, to four demand points, 1, 2, 3 and 4. It also shows the stock held at each supply point and the stock required at each demand point. A minimum cost solution is required.

	1	2	3	4	Supply
A	23	32	25	30	15
B	34	29	32	28	17
C	37	27	31	33	23
D	34	28	29	35	16
Demand	18	20	14	19	

Table 1

Table 2 shows an initial solution given by the north-west corner method. Table 3 shows some of the improvement indices for this solution.

	1	2	3	4
A	15			
B	3	14		
C		6	14	3
D				16

Table 2

	1	2	3	4
A	X	14	3	6
B	X	X		
C		X	X	X
D	0	–1		X

Table 3

(a) Calculate the shadow costs and enter the missing improvement indices into Table 3 in your answer book.

(2)

(b) Taking the most negative improvement index to indicate the entering cell, use the stepping-stone method once to obtain an improved solution. You must make your method clear by
- showing your route
- stating the entering cell and exiting cell.

(3)

(c) Determine whether your current solution is optimal, giving a reason for your answer.

(3)

The stock held at supply point A is increased to 20

For the transportation algorithm to be used, Table 1 needs to be modified.

(d) (i) Explain why Table 1 needs to be modified.

(ii) Describe how Table 1 should be modified.

You do no not need to solve this problem.

(3)

(Total for Question 1 is 11 marks)

2. Four workers, A, B, C and D, are to be assigned to four tasks, 1, 2, 3 and 4. Each worker must be assigned to just one task and each task must be done by just one worker. The total cost of assigning the workers to the tasks is to be minimised.

The cost, in pounds, of assigning each worker to each task is shown in the table below.

	1	2	3	4
A	29	15	32	27
B	33	25	30	31
C	40	43	37	34
D	30	20	27	37

Part of a linear programming formulation to model this problem is as follows:

$$\text{Let } x_{ij} = \begin{cases} 1 & \text{if worker } i \text{ does task } j \\ 0 & \text{otherwise} \end{cases}$$

Where $i \in \{A, B, C, D\}$ and $j \in \{1, 2, 3, 4\}$

(a) Write down the constraints of this linear programming problem.

(2)

(b) Write down the objective function of the linear programming problem.

(1)

(c) Reducing rows first, use the Hungarian algorithm to obtain an allocation that minimises the total cost of assigning all four workers to tasks. You must make your method clear and show the table after each stage.

(6)

(Total for Question 2 is 9 marks)

3. Susie needs to buy a new boat. She has narrowed her choice down to one of three boats, A, B and C. From experience, Susie knows at the end of each year whichever boat she buys it will need repairing. For each of the three boats there is the option of buying annual insurance and, if bought, this insurance will cover all the repair costs at the end of the year. The cost, each year, of the insurance is £2000 for boat A, £2050 for boat B and £2100 for boat C.

If Susie does not buy insurance then she will have to pay the annual repair bill herself. At the end of the year the boat will require one of two types of repair. These two types can be classified as either 'minor' or 'major'. The repair costs and corresponding probabilities are shown in the table below.

Boat	Cost of minor repair	Probability of minor repair	Cost of major repair	Probability of major repair
A	1880	$\frac{4}{5}$	2350	$\frac{1}{5}$
B	2020	$\frac{3}{4}$	2300	$\frac{1}{4}$
C	1620	$\frac{2}{3}$	2670	$\frac{1}{3}$

(a) Draw a decision tree to model Susie's possible decisions and the possible outcomes.

(6)

(b) Given that Susie wishes to minimise her potential expenditure, calculate her optimal EMV of annual repair costs and state the optimal strategy indicated by the decision tree.

(2)

(Total for Question 3 is 8 marks)

4.

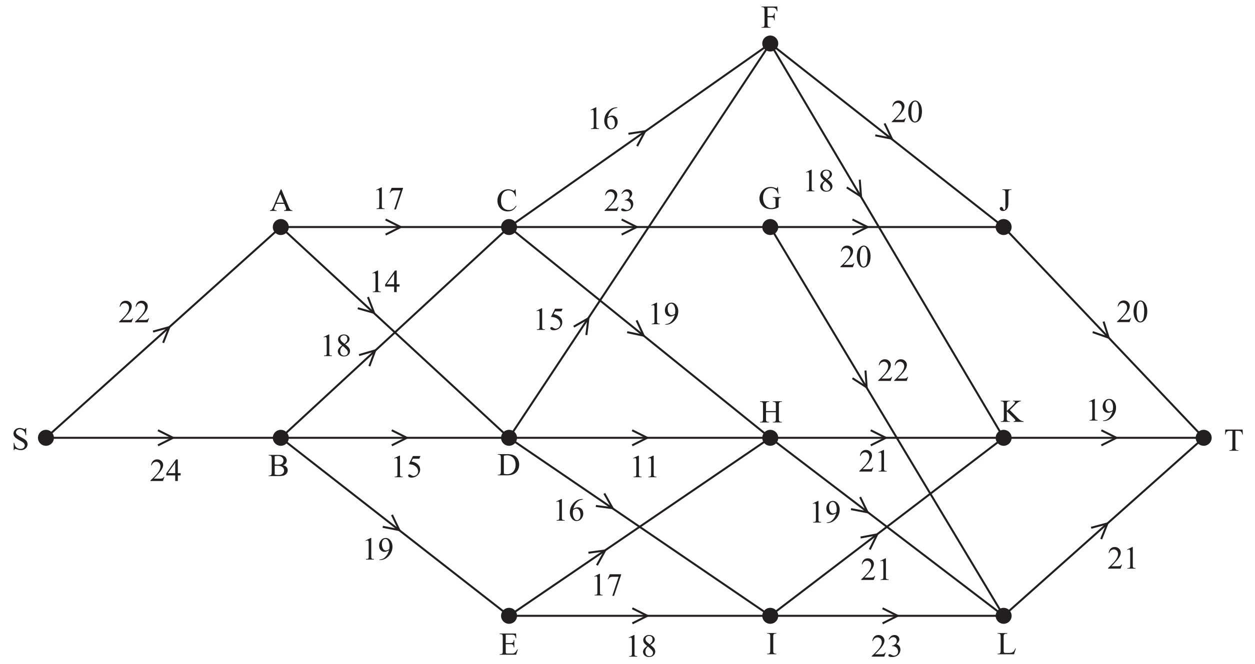

Figure 1

A company needs to transport goods from its base at S to its depot at T. The directed network shown in Figure 1 represents the roads from S to T. The number on each arc represents the maximum weight limit, in tonnes, for the corresponding road. The company needs to find the maximum possible weight of a loaded truck that can make the journey from S to T.

(a) Write down the type of dynamic programming problem that the company needs to solve.

(1)

(b) Use dynamic programming to find

(i) the maximum weight of a loaded truck that can make the journey from S to T

(ii) the route that the truck could take.

(12)

Due to engineering works, all roads leading out of B are closed.

(c) (i) State the new maximum weight of a loaded truck that can make the journey from S to T.

(ii) State the corresponding route.

(2)

(Total for Question 4 is 15 marks)

5.

		Player B	
		Option X	Option Y
Player A	Option P	–3	3
	Option Q	2	1
	Option R	3	–2
	Option S	0	–1

A two person zero-sum game is represented by the pay-off matrix for player A shown above.

(a) Determine the number of points player B will gain if player A plays option R and player B plays option Y.
(1)

(b) Explain, with justification, why this matrix may be reduced to a 3×2 matrix by removing option S from player A's choices.
(2)

(c) Verify that there is no stable solution to this reduced game.
(3)

Player A intends to make a random choice between her options P, Q and R, choosing option P with probability p_1, option Q with probability p_2 and option R with probability p_3

She formulates the following linear programming problem so that she can find the optimal values of p_1, p_2 and p_3 using the Simplex algorithm.

Maximise $P = V$

subject to $V \leqslant 5p_2 + 6p_3$
$V \leqslant 6p_1 + 4p_2 + p_3$
$p_1 + p_2 + p_3 \leqslant 1$
$p_1 \geqslant 0, p_2 \geqslant 0, p_3 \geqslant 0, V \geqslant 0$

(d) (i) Show how she obtained the expressions

$$5p_2 + 6p_3 \quad \text{and} \quad 6p_1 + 4p_2 + p_3$$

(ii) Explain why V cannot exceed either of these two expressions.
(3)

(e) Explain why the constraint $p_1 + p_2 + p_3 \leqslant 1$ is needed.
(1)

The Simplex algorithm is used to solve the linear programming problem.

The optimal value of p_1 is $\frac{1}{7}$ and the optimal value of p_3 is 0

(f) (i) Calculate the optimal value of p_2

(ii) Calculate the value of the game to player B.
(3)

(Total for Question 5 is 13 marks)

6. Sequences $\{x_n\}$ and $\{y_n\}$ for $n \in \mathbb{N}$, are defined by

$$x_{n+1} = 4y_n - 2 \quad \text{and} \quad y_{n+1} = 0.16x_n - 4$$

where $x_1 = -50.8$ and $y_1 = -11.52$

(a) Show that

$$x_{n+1} - 0.64x_{n-1} = k$$

where k is a constant to be determined.

(2)

(b) Find the solution of this second order recurrence relation to obtain an expression for x_n in terms of n.

(8)

(c) Hence find an expression for y_n in terms of n.

(2)

(d) State the limiting value of

(i) $\{x_n\}$

(ii) $\{y_n\}$

(1)

(Total for Question 6 is 13 marks)

7.

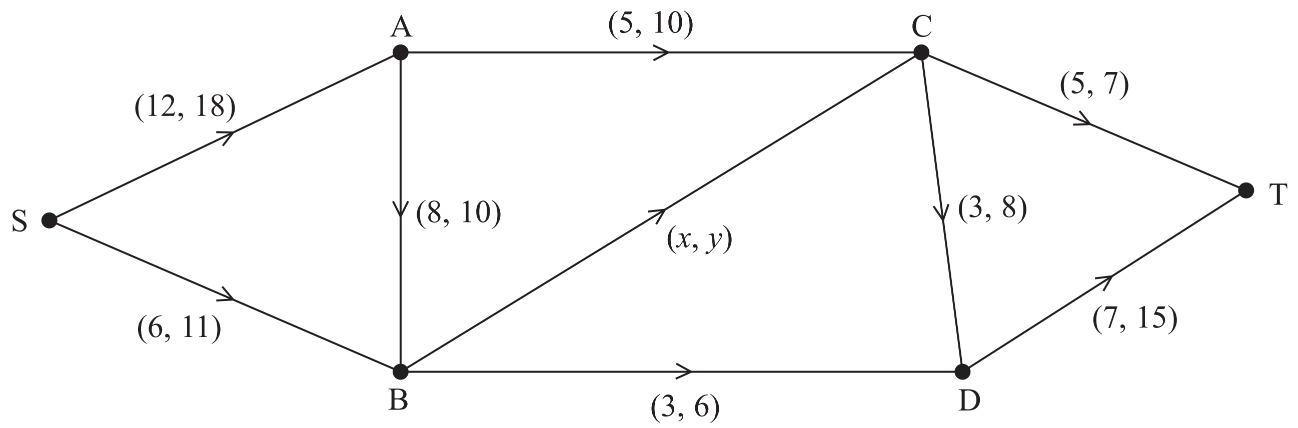

Figure 2

Figure 2 shows a capacitated, directed network. The numbers on each arc (l, u) represent the lower capacity (l) and upper capacity (u) of that arc.

(a) Find the range of possible values of the flow in arc BC, giving reasons for your answer.

(4)

(b) Write down the three constraints on the values of x and y.

(2)

(Total for Question 7 is 6 marks)

TOTAL FOR PAPER IS 75 MARKS