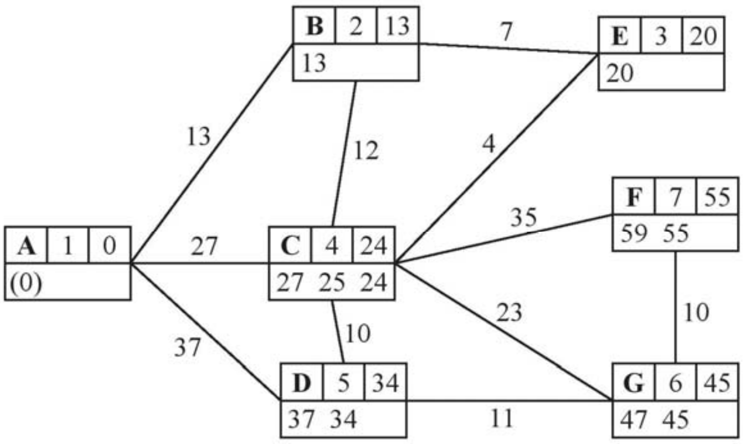
Paper 2 Option K

Decision Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)		M1 A1 A1	1.1b 1.1b 1.1b
	Path: ABECDGF	A1	1.1b
	Length: 55 (metres)	A1ft	1.1b
		(5)	
(b)	AB + DG = 13 + 11 = 24 ←	M1	1.1b
	A(BEC)D + B(ECD)G = 34 + 32 = 66	A1	1.1b
	A(BECD)G + B(EC)D = 45 + 21 = 66	A1	1.1b
	Repeat arcs: AB, DG	A1ft	2.2a
		(4)	
(c)	Length = 189 + 24 = 213 (metres)	B1ft	1.1b
		(1)	
(d)	$189 + x + 34 = 213 + 2x$	M1	3.1b
	$x = 10$ so BG is 10 m	A1	1.1b
		(2)	
			(12 marks)

Notes:

(a)
M1: For a larger number replaced by a smaller one in the working values boxes at C, D, F or G
A1: For all values correct (and in correct order) at A, B, C and D
A1: For all values correct (and in correct order) at E, F & G
A1: For the correct path
A1ft: For 55 or ft their final value at F

(b)
M1: For 3 correct pairings of the four odd nodes (A,B, D & G)
A1: At least two pairings and totals correct
A2: All three pairings and totals correct
A3ft: Selecting their shortest pairing, and stating that these arcs should be repeated

Question 1 notes continued:	
(c)	
B1ft:	For 213 or 189 + their shortest repeat
M1:	For translating the information in the question in to an equation involving x, $2x$ and 34
A1:	For a correct equation leading to BG = 10 (m)

Question	Scheme	Marks	AOs
2	Objective line drawn or at least two vertices tested	M1	3.1a
	For solving y = $4x$ and $8x + 7y = 560$ to find the exact co-ordinate of the optimal point, must reach either $x =$ or $y =$	M1	1.1a
	$x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$	A1	1.1b
	Finding at least two points with integer co-ordinates from $(15 \pm 1,\ 63 \pm 2)$	M1	1.1b
	Testing at least two points with integer co-ordinates	M1	1.1b
	$x = 15$ and $y = 63$	A1	2.2a
	So the teacher should buy 15 pens and 63 pencils	A1ft	3.2a
		(7 marks)	

Notes:

M1: Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

M1: Solving simultaneous equations

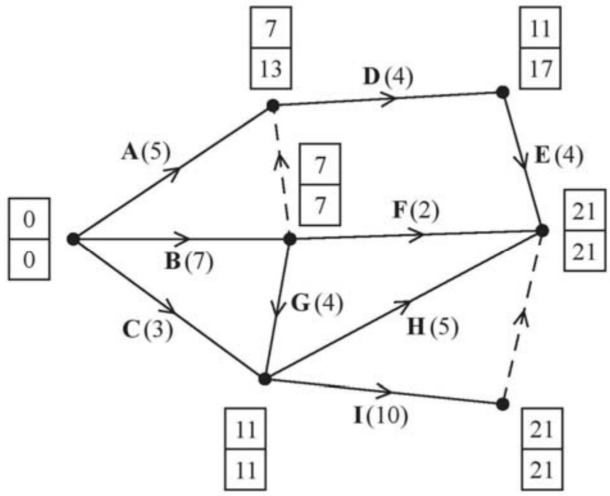
A1: cao

M1: Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

M1: Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

A1: cao – deducing from tests which integer solution is both valid and optimal

A1ft: Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td rowspan="4">3(a)(b)</td><td>
</td><td>M1
A1
A1</td><td>1.1b
1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td>The number(s) at the end of activity E indicate this project can be completed in 21 days

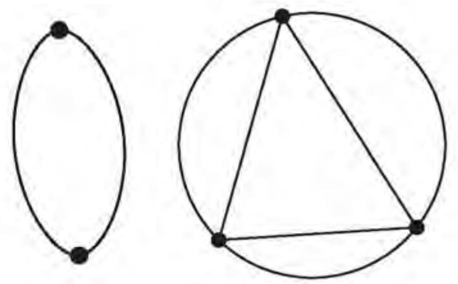
Critical activities: B, G, I</td><td>M1
A1

A1ft

A1</td><td>2.1
1.1b

2.2a

1.1b</td></tr>
<tr><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4" align="right">(7 marks)</td></tr>
<tr><td colspan="4">Notes:</td></tr>
<tr><td colspan="4">M1: At least 5 activities and one dummy, one start
A1: A,B,C,D,F,G and first dummy correct
A1: E,H,I correct, second dummy correct and one finish</td></tr>
<tr><td colspan="4">M1: All boxes completed, number generally increasing L to R (condone one “rogue”)
A1: All values cao
A1: Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes)
A1: Critical activities correct</td></tr>
</table>

Question	Scheme	Marks	AOs
4(a)	e.g. a graph cannot contain an odd number of odd nodes e.g. number of arcs $= \dfrac{1+3+4+4+5}{2} = 8.5 \notin \mathbb{Z}$	B1	2.4
		(1)	
(b)(i)	$\left(2^{2x}-1\right)+\left(2^{x}\right)+\left(x+1\right)+\left(2^{x+1}-3\right)+\left(11-x\right)=2(18)$	M1	1.1b
	$2^{2x}+3\left(2^{x}\right)-28=0 \Rightarrow x=\ldots$	M1	1.1b
	$\left(2^{x}+7\right)\left(2^{x}-4\right)=0 \Rightarrow x=2$	A1	1.1b
		(3)	
(b)(ii)	The order of the nodes are 9, 15, 3, 4, 5	M1	2.1
	Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes	A1 A1	2.4 2.2a
		(3)	
(c)		M1 A1	2.5 2.2a
		(2)	
		(9 marks)	

Notes:

(a)
B1: Explanation referring to need for an even number of odd nodes oe

(b)
M1: Forming an equation involving the orders of the 5 odd nodes and 2(18)
M1: Simplifies to a quadratic in 2^x and attempts to solve
A1: 2 cao
M1: Construct an argument involving the order of the 5 nodes
A1: Explanation considering the number of odd nodes
A1: Deduction that therefore it is neither Eulerian nor semi-Eulerian

(c)
M1: Interprets mathematical language to construct a disconnected graph
A1: Deduce a correct graph

Question	Scheme	Marks	AOs
5	Minimise $(C =)\ 25x+35y$	B1	3.3
	Subject to: $(500x+800y \geqslant 150\ 000 \Rightarrow)\ 5x+8y \geqslant 1500$	B1	3.3
	$\frac{7}{20}(x+y) \leqslant x \leqslant \frac{13}{20}(x+y)$	M1 M1	3.3 3.3
	Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ $x, y \geqslant 0$	A1	1.1b
			(5 marks)

Notes:

B1: A correct objective function + minimise
B1: Translate information in to a correct inequality
M1: For translating the information given into the LHS inequality
M1: For translating the information given in to the RHS inequality
A1: Simplifying to the correct inequalities

Decision Mathematics 2 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
6	$\begin{array}{cc} & \begin{array}{cccccc} P & Q & R & S & T & X \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 32 & 32 & 35 & 34 & 33 & 40 \\ 28 & 35 & 31 & 37 & 40 & 40 \\ 35 & 29 & 33 & 36 & 35 & 40 \\ 36 & 30 & 34 & 33 & 35 & 40 \\ 30 & 31 & 29 & 37 & 36 & 40 \\ 29 & 28 & 32 & 31 & 34 & 40 \end{pmatrix} \end{array}$	B1	1.1b
	Reducing rows and then columns $\begin{array}{cc} & \begin{array}{cccccc} P & Q & R & S & T & X \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 0 & 3 & 2 & 1 & 8 \\ 0 & 7 & 3 & 9 & 12 & 12 \\ 6 & 0 & 4 & 7 & 6 & 11 \\ 6 & 0 & 4 & 3 & 5 & 10 \\ 1 & 2 & 0 & 8 & 7 & 11 \\ 1 & 0 & 4 & 3 & 6 & 12 \end{pmatrix} \end{array}$ then $\begin{array}{cc} & \begin{array}{cccccc} P & Q & R & S & T & X \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 7 & 3 & 7 & 11 & 4 \\ 6 & 0 & 4 & 5 & 5 & 3 \\ 6 & 0 & 4 & 1 & 4 & 2 \\ 1 & 2 & 0 & 6 & 6 & 3 \\ 1 & 0 & 4 & 1 & 5 & 4 \end{pmatrix} \end{array}$	M1 A1	1.1b 1.1b
	e.g. augment by 1 then augment by 1 $\begin{array}{cc} & \begin{array}{cccccc} P & Q & R & S & T & X \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 1 & 1 & 3 & 0 & 0 & 0 \\ 0 & 7 & 2 & 6 & 10 & 3 \\ 6 & 0 & 3 & 4 & 4 & 2 \\ 6 & 0 & 3 & 0 & 4 & 1 \\ 2 & 3 & 0 & 6 & 6 & 3 \\ 1 & 0 & 3 & 0 & 4 & 3 \end{pmatrix} \end{array}$ followed by $\begin{array}{cc} & \begin{array}{cccccc} P & Q & R & S & T & X \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 2 & 2 & 3 & 1 & 0 & 0 \\ 0 & 7 & 1 & 6 & 9 & 2 \\ 6 & 0 & 2 & 4 & 3 & 1 \\ 6 & 0 & 2 & 0 & 3 & 0 \\ 3 & 4 & 0 & 7 & 6 & 3 \\ 1 & 0 & 2 & 0 & 3 & 2 \end{pmatrix} \end{array}$	M1 A1ft M1 A1ft A1	1.1b 1.1b 1.1b 1.1b 1.1b
	A – T, B – P, C – Q, (D –), E – R, F – S	A1	2.2a
			(9 marks)

Notes:

B1: cao – introducing a dummy task and appropriate value
M1: Simplifying the initial matrix by reducing rows and then columns
A1: cao
M1: Develop an improved solution – need to see Double covered +e; one uncovered –e ; and one single covered unchanged. 4 lines to 5 lines needed
A1ft: ft on their previous table – no errors
M1: Finding the optimal solution – need to see one double covered +e; one uncovered –e; and one single covered unchanged. 5 lines needed to 6 lines needed (so getting to the optimal table)
A1ft: ft on their previous table – no errors
A1: cso on final table (so must have scored all previous marks)
A1: cso – this mark is dependent on all M marks being awarded – to deduce the optimal allocation from the location of zeros in the table

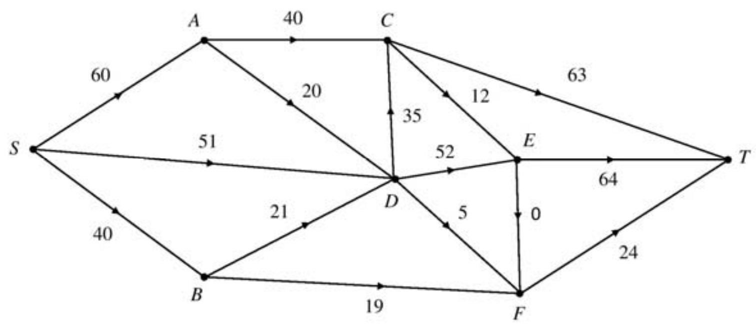
Question	Scheme	Marks	AOs
7(a)	16, 22, 29	B1	1.1b
		(1)	
(b)	$u_{n+1} = u_n + n + 1$	B1	3.3
		(1)	
(c)	As $u_{n+1} = u_n + p(n) \Rightarrow u_n = \lambda n^2 + \mu n + \phi$ and attempt to solve with $n = 1, 2, 3$	M1	1.1b
	$u_n = \frac{1}{2}n(n+1)+1$	A1	1.1b
	20 101 (regions)	A1ft	1.1b
		(3)	
			(5 marks)

Notes:

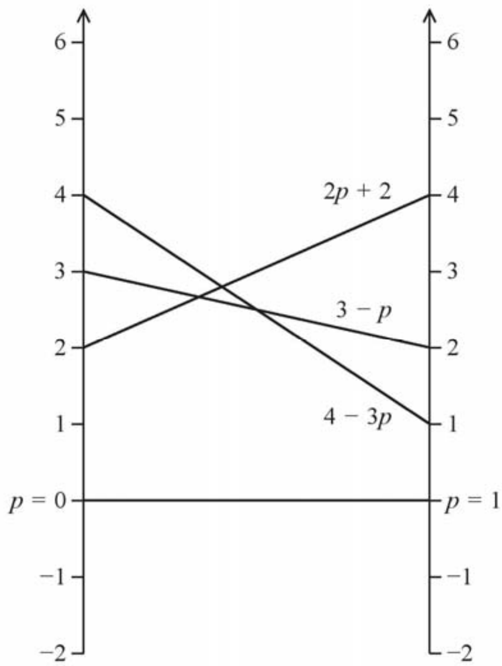
(a)
B1: cao

(b)
B1: Translating problem to mathematical model - correct recurrence relation needed

(c)
M1: An attempt to solve the recurrence relation to determine maximum number of regions
A1: cao
A1ft: Substitution of n = 200 into their quadratic u_n expression

Question	Scheme	Marks	AOs
8(a)	Corridors must be one-way	B1	3.4
		(1)	
(b)	e.g. $55 + x + 40 = 63 + 54 + 24$ **or** $7 + y = 54 + 5$	M1	2.4
	$x = 46$ $y = 52$	A1 A1	1.1b 1.1b
		(3)	
(c)	**(i)** SACET (= 5) SDFET (= 5)	M1 A1	1.1b 1.1b
	(ii) Students must choose SACET, as they cannot travel from F to E	A1	2.2a
		(3)	
(d)		B1	1.1b
		(1)	
(e)	Use of max-flow min-cut theorem	M1	2.1
	Identification of cut through AC, DC, DE, (EF), FT = 151 value of flow = 151	A1	3.1a
	Therefore it follows that flow is optimal	A1	2.2a
		(3)	
(f)	Consider increasing capacity of arcs in minimum cut	B1	2.1
	Explanation based on a valid argument, such as: • increasing the capacity of any arc other than FT would not increase the flow by more than 1, as total capacity directly in to T is only 152 • increasing the capacity on FT could increase the total flow by 16 (increased flow along SAD, SD and SBD could all be directed through DF to F)	B1	2.4
	Therefore school should choose to widen FT, which could increase the flow through the network by 16	B1	2.2a
		(3)	
			(14 marks)

Question 8 notes:	
(a)	
B1:	Explanation of assumption to use this model
(b)	
M1:	Either a correct equation, or explanation that flow in = flow out
A1:	cao
A1:	cao
(c)	
M1:	One flow augmenting route found from S to T
A1:	Two correct flow augmenting routes 5+
A1:	Deduce that SACET must be used as students cannot travel from F to E as route is one-way
(d)	
B1:	A consistent flow pattern = 151
(e)	
M1:	Constructing argument based on max-flow min-cut theorem
A1:	Use appropriate process of finding a minimum cut – cut + value correct
A1:	Correct deduction that the flow is maximal
(f)	
B1	Constructing an argument based on arcs in the minimum cut
B1	Detailed explanation as to why choosing anything other than FT does not help
B1	Correct deduction and correct increase in flow of 16

Question	Scheme	Marks	AOs
9(a)	Row minima: 1, 2 max is 2 Column maxima: 4, 4, 3 min is 3	M1 A1	1.1b 1.1b
	Row maximin (2) $\neq$ Column minimax (3) so not stable	A1	2.4
		(3)	
(b)	Let A play strategy 1 with probability p and strategy 2 with probability $1 - p$, and using this to get at least one equation in p	M1	3.3
	Then if B plays strategy 1, A's gains are $4p + 2(1 - p) = 2p + 2$ If B plays strategy 2, A's gains are $p + 4(1 - p) = 4 - 3p$ If B plays strategy 3, A's gains are $2p + 3(1 - p) = 3 - p$ 	A1 A1	1.1b 1.1b
	Intersection of $2p + 2$ and $3 - p$ occurs where $p = \frac{1}{3}$	dM1 A1ft	1.1b 1.1b
	Therefore player A should play strategy 1 $\frac{1}{3}$ of the time and play strategy 2 $\frac{2}{3}$ of the time	A1ft	3.2a
	The value of the game to player A is $2\frac{2}{3}$	A1	1.1b
		(9)	
		(12 marks)	

Question 9 notes:	
(a)	
M1:	Finding row minimums and column maximums – condone one error
A1:	Row minima and column maxima correct
A1:	Explanation involving $2 \neq 3$ and a conclusion
(b)	
M1:	Translating situation into model by defining variables and constructing at least one equation
A1:	One row correct
A1:	All three rows correct
M1:	Axes correct, at least one line correctly drawn for their expression
A1:	Correct graph
M1:	Using their probability expectation graph to find the probability by equating their two correct expressions and attempting to solve as far as $p =$
A1ft:	ft on their optimal intersection
A1ft:	Interpret their value of p in the context of the question – must refer to play, player A
A1:	cao