

## Pearson Edexcel Level 3 GCE

# Thursday 16 May 2019

Afternoon

Paper Reference **8FM0-28**

## Further Mathematics

**Advanced Subsidiary**

**Further Mathematics options**

**28: Decision Mathematics 2**

**(Part of option K only)**

### You must have:

Mathematical Formulae and Statistical Tables (Green), calculator,  
D2 Answer Book (enclosed)

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for each question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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1. Three workers, A, B and C, are each to be assigned to one of four tasks, P, Q, R and S.

Each worker must be assigned to at most one task, and each task must be done by at most one worker.

The amount, in pounds, that each worker will earn while assigned to each task is shown in the table below.

	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

The Hungarian algorithm is to be used to find the maximum total amount that can be earned by the three workers.

- (a) Explain how the table should be modified. (2)
- (b) (i) Reducing rows first, use the Hungarian algorithm to obtain an allocation which maximises the total earnings.
- (ii) Explain how any initial row and column reductions were made and also how you determined if the table was optimal at each stage. (7)

**(Total for Question 1 is 9 marks)**

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2. (a) Find the general solution of the recurrence relation

$$u_{n+1} = 3u_n + 2^n \quad n \geq 1 \quad (4)$$

- (b) Find the particular solution of this recurrence relation for which  $u_1 = u_2$  (2)

**(Total for Question 2 is 6 marks)**

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3.

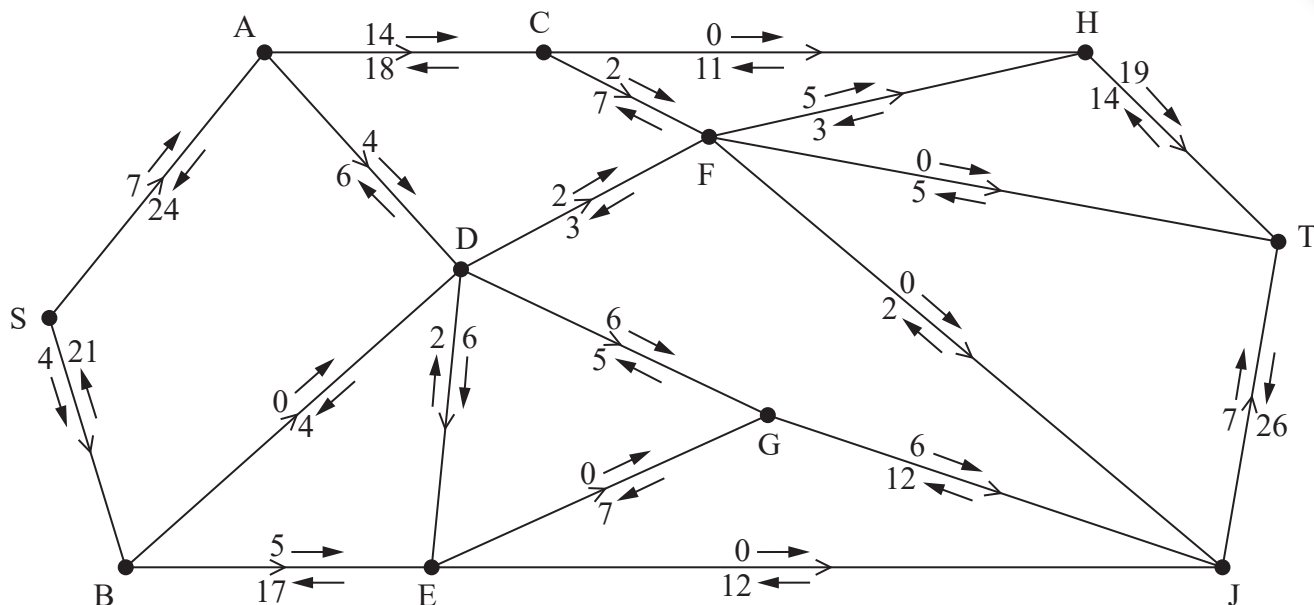


Figure 1

Alexa is monitoring a system of pipes through which fluid can flow from the source, S, to the sink, T. Currently, fluid is flowing through the system from S to T.

Alexa initialises the labelling procedure for this system, and the excess capacities and potential backflows are shown on the arrows either side of each arc, as shown in Figure 1.

- (a) State the value of the initial flow. (1)
- (b) Explain why arcs DF and DG can never both be full to capacity. (1)
- (c) Obtain the capacity of the cut that passes through the arcs AC, AD, BD, DE, EG and EJ. (1)
- (d) Use the labelling procedure to find a maximum flow through the network. You must list each flow-augmenting route you use, together with its flow. (3)
- (e) Use your answers to part (d) to find a maximum flow pattern for this system of pipes and draw it on Diagram 1 in the answer book. (1)
- (f) Prove that the answer to part (e) is optimal. (3)

(Total for Question 3 is 10 marks)

4. The table below gives the pay-off matrix for a zero-sum game between two players, Aljaz & Brendan. The values in the table show the pay-offs for Aljaz.

		Brendan		
		Option X	Option Y	Option Z
Aljaz	Option P	-6	-1	2
	Option Q	5	4	-7
	Option R	5	6	3

- (a) (i) Show that this game is stable.  
(ii) State the value of this game to Brendan. (3)

Option R is removed from Aljaz's choices and the reduced game, with option R removed, is no longer stable.

- (b) Find the best strategy for Aljaz in this reduced game, defining any variable you use. (7)
- (c) Explain why Brendan should never play option Y (1)

Let Brendan play option X with probability  $q$

- (d) (i) Explain why  $q$  satisfies the equation  $6q - 2(1 - q) = 1.6$   
(ii) Hence find the best strategy for Brendan in this reduced game. (4)

(Total for Question 4 is 15 marks)

**TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS**

**END**

Please check the examination details below before entering your candidate information

Candidate surname	Other names
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**Pearson Edexcel  
Level 3 GCE**

Centre Number

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Candidate Number

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(Part of option K only)

**Answer Book**  
Do not return the question paper with the answer book.

Total Marks

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Turn over ►



1. (a)

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	P	Q	R	S
A	32	40	37	42
B	29	32	35	41
C	37	33	39	40

*You may not need to use all of these tables  
You may not need to use all the rows and columns*

	P	Q	R	S	
A					
B					
C					

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	P	Q	R	S	
A					
B					
C					

	P	Q	R	S	
A					
B					
C					





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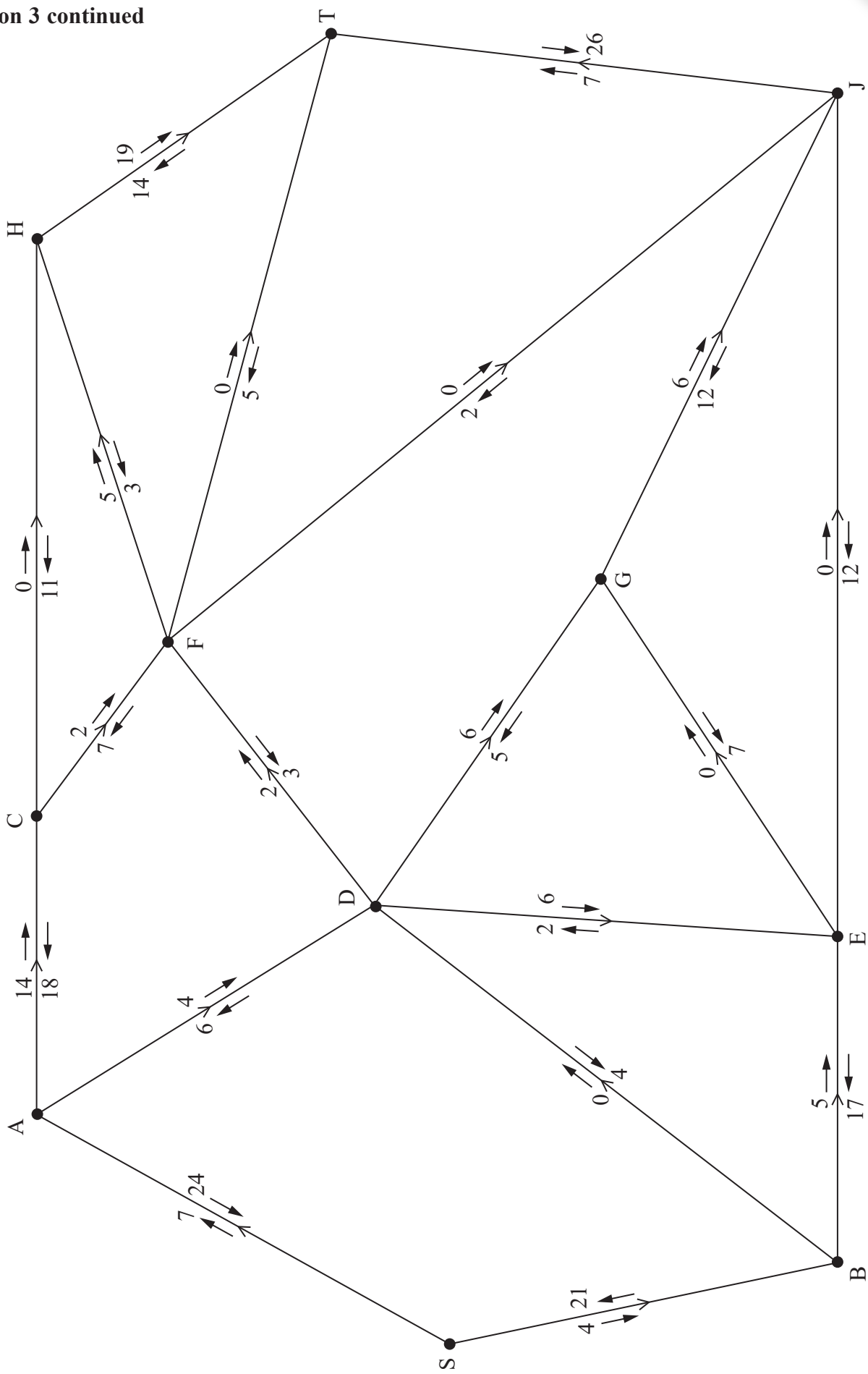
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Question 3 continued



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Question 3 continued

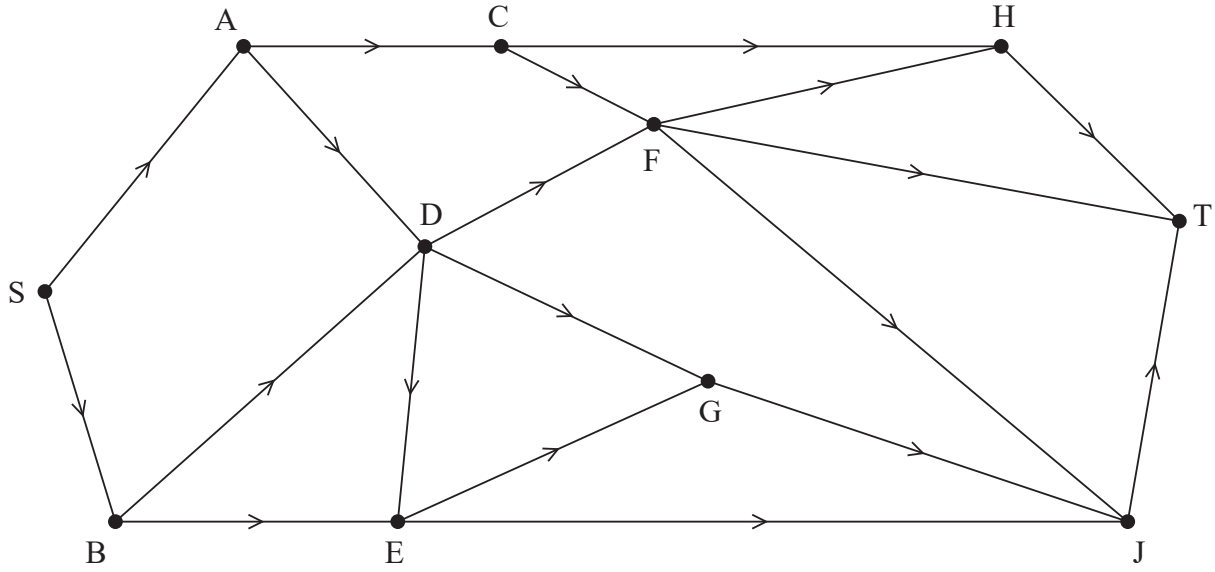
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**Question 3 continued**



**Diagram 1**

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**(Total for Question 3 is 10 marks)**



P 6 1 6 4 0 A 0 9 1 2

4.

Brendan

	Option X	Option Y	Option Z
Option P	-6	-1	2
Option Q	5	4	-7
Option R	5	6	3

Aljaz

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**Question 4 continued**

A series of horizontal lines for writing an answer.

**(Total for Question 4 is 15 marks)**

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**TOTAL FOR DECISION MATHEMATICS 2 IS 40 MARKS**



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