Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate

MATHEMATICS (PRINCIPAL) **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2018**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

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1 A geometric progression $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 32$ and $u_{n+1} = 0.75u_n$ for $n \geqslant 1$.

(i) Find u_5. [2]

(ii) Find $\sum_{n=1}^{\infty} u_n$. [2]

2 **(i)** Express $2x^2 + 6x + 5$ in the form $p(x+q)^2 + r$. [3]

(ii) State the equation of the line of symmetry of the curve $y = 2x^2 + 6x + 5$. [1]

(iii) Find the value of the constant k for which the line $y = k - 2x$ is a tangent to the curve $y = 2x^2 + 6x + 5$. [3]

3 Solve the equation $6^{2x-1} = 3^{x+2}$, giving your answer in the form $x = \frac{\ln a}{\ln b}$ where a and b are integers. [5]

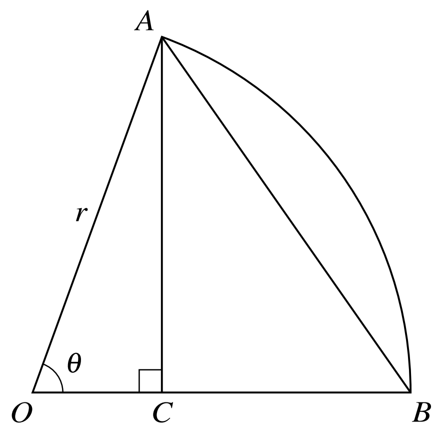
4 Solve the equation $x + 2\sqrt{x} - 6 = 0$, giving your answer in the form $x = c + d\sqrt{7}$ where c and d are integers. [6]

5 The complex numbers u and v are given by $u = 3 + 2\text{i}$ and $v = 1 + 4\text{i}$.

(i) Given that $au^2 + bv^* = 7 + 36\text{i}$ find the values of the real constants a and b. [5]

(ii) Show the points representing u and v on an Argand diagram and hence sketch the locus given by $|z - u| = |z - v|$. Find the point of intersection of this locus with the imaginary axis. [4]

6



The diagram shows a sector AOB of a circle, centre O and radius r. Angle AOB is θ radians. The point C lies on OB, and AC is perpendicular to OB. The area of the triangle AOC is equal to the area of the segment bounded by the chord AB and the arc AB.

(i) Show that $\theta = \sin\theta(1 + \cos\theta)$. [4]

The equation $\theta = \sin\theta(1 + \cos\theta)$ has only one positive root.

(ii) Use an iterative process based on this equation to find the value of the root correct to 3 significant figures. Use a starting value of 1 and show the result of each iteration. Use a change of sign to verify that the value you have found is correct to 3 significant figures. [5]

7 A curve is given parametrically by $x = t^2 + 1$, $y = t^3 - 2t$ where t is any real number.

(i) Show that the equation of the normal to the curve at the point where $t = 2$ can be written in the form $2x + 5y = 30$. [5]

(ii) Show that this normal does not meet the curve again. [5]

8 **(i)** Use integration by parts twice to show that

$$\int e^x \sin x \, dx = \tfrac{1}{2}e^x(\sin x - \cos x) + c. \qquad [6]$$

(ii) Hence find the equation of the curve which passes through the point (0, 2) and for which $\dfrac{dy}{dx} = e^x \sin x$. [2]

Questions 9 and 10 are printed on the next page.

9 In this question, x denotes an angle measured in degrees.

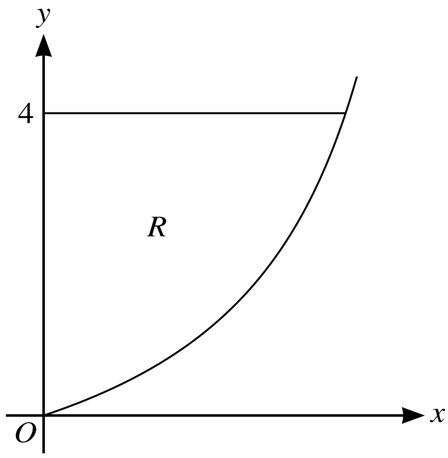
(i) Express $4\sin(2x + 30°) + 3\cos 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Give full details of the sequence of transformations which maps the graph of $y = \cos x$ onto the graph of $y = 4\sin(2x + 30°) + 3\cos 2x$. [4]

(iii) Find the smallest positive value of x that satisfies the equation $4\sin(2x + 30°) + 3\cos 2x = 6$. [4]

10 **(i)** By using the substitution $u = 3 - 2x$, or otherwise, show that $\int_0^1 \left(\frac{4x}{3-2x}\right)^2 \mathrm{d}x = 16 - 12\ln 3$. [7]

(ii)



The diagram shows the region R, which is bounded by the curve $y = \frac{4x}{3-2x}$, the y-axis and the line $y = 4$. Find the exact volume generated when the region R is rotated completely around the x-axis. [3]