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MATHEMATICS **9794/02**

Paper 2 Pure Mathematics 2 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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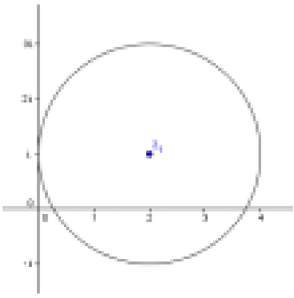
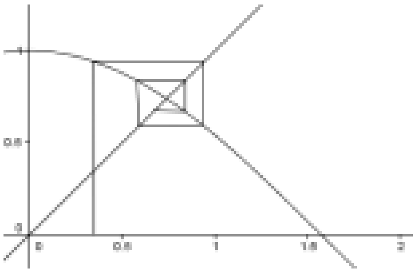
1 **(i)**	f(–2) = –12	M1 A1 [2]	Substitute $x = -2$, or any other complete method – must get as far as attempting the remainder but allow no more than 2 errors If using inspection then allow M1 for $(x + 2)(x^2 - 2x + k) - 2k$ Obtain –12 (no isw if then given as 12 or if given as $^{-12}/_{(x+2)}$) Must be identified as remainder so A0 if just left at bottom of division attempt
(ii)	12	B1FT [1]	FT on their **(i)**
2	$3^x = {}^5/_4$ $x = \log_3({}^5/_4)$	B1* M1d* M1d* A1 [4]	State $3^x = {}^5/_4$ Allow using logs before rearranging, as long as valid method to deal with $\log(4 \times 3^x)$ Take logarithms and apply at least one log rule correctly Rearrange to make x the subject Obtain correct answer aef Allow BOD if no base specified ISW decimal answer but not subsequent incorrect log work, such as $\log({}^5/_4)/\log(3) = \log({}^5/_{12})$
3	$\log_{10} y = 2x + 4$ $y = 10^{2x+4}$ $= 10^{2x} \times 10^4$ $= 10000 \times 100^x$ **AG** **OR** $y = 10000 \times 100^x$ $\log_{10} y = \log_{10}10000 + \log_{10}100^x$ $\log_{10} y = 2x + 4$ Conclude convincingly	M1 A1 M1 A1 [4]	State equation of form $\log_{10} y = mx + c$ State $\log_{10} y = 2x + 4$ Base 10 must be seen, or implied by later work Attempt correct process to remove logs Obtain $y = 10^{2x} \times 10^4$ and hence $y = 10000 \times 100^x$ M1 – take logs of both sides M1 – use one correct log rule A1 – obtain $\log_{10} y = 2x + 4$ A1 – relate to $y = mx + c$
4 **(i)**	$\lvert z_1\rvert = \sqrt{5}$ $\lvert z_2\rvert = 5$ $z_1 + z_2 = 5 + 5i$ $\lvert z_1 + z_2\rvert = \sqrt{50}$ $\sqrt{5} + 5 > \sqrt{50}$	B1 M1 A1 A1 [4]	Both correct Attempt $z_1 + z_2$ Could be implied by attempt at $\lvert z_1 + z_2\rvert$ Obtain $\sqrt{50}$ oe Conclude by approximating to sufficient accuracy or comparing surds – A0 if no clear comparison Could also use geometrical argument

	(ii)		B1 B1 [2]	Circle Centre at 2 + i and radius of 2 soi Circle should be approximately correct i.e. have the y-axis as a tangent, and not pass through the origin
5	(i)	$\frac{3(x+1)+(x+2)}{(x+2)(x+1)} = \frac{4x+5}{x^2+3x+2}$ **OR** A(x+1) + B(x + 2) = 4x+ 5 so A = 3 and B = 1.	M1 A1 [2]	Attempt to add fractions using common denominator Simplify to obtain given answer M1 – use partial fractions on RHS A1 – obtain given answer
	(ii)	$-\frac{3}{(x+2)^2} - \frac{1}{(x+1)^2}$	M1 A1 A1 [3]	Differentiate both terms on the LHS, or any other valid method Obtain one correct term Obtain fully correct $f'(x)$ Quotient rule: M1 – attempt quotient rule A1 – correct unsimplified expression A1 – correct simplified expression
	(iii)	Denominators always +ve as $(x + k)^2 > 0$ Numerators always –ve, and $^{-ve}/_{+ve}$ is -ve	M1 A1 [2]	State, or imply, that “decreasing” implies $f'(x)<0$, and make some attempt to use this Conclude convincingly that $f'(x)<0$ for all x (CWO, A0 if incorrect $f'(x)$)
6	(i)	Angle AOB $= \cos^{-1}\frac{16+6+20}{\sqrt{38\times 84}}$ $= 42.0°$	M1 M1 M1 A1 [4]	Attempt $a.b$ for $\pm OA$ and $\pm OB$ (at least 2 elements correct) Use correct formula for their vectors Attempt evaluation, with correct two vectors Obtain 42.0° (allow 42°) or 0.733 rad If using cosine rule, then M1 – attempt sides (at least 2 correct) M1 – attempt cosine rule M1 – rearrange to attempt angle A1 – obtain 42.0°

	(ii)	$BA = -6\mathbf{i} + \mathbf{j} + \mathbf{k}$ $\lvert BA\rvert = \sqrt{38}$ $\lvert BA\rvert = \lvert OA\rvert$ hence isosceles ($\neq \lvert OB\rvert$ not nec)	B1 M1 A1 [3]	State correct *BA* or *AB* Find one side length or one angle other than those found in part **(i)** If *BA* or *AB* has been stated then sufficient to just state $\sqrt{38}$ If *BA* or *AB* has not been stated then a minimum of $\sqrt{(36+1+1)}$ must be seen Conclude convincingly NB Angles and sides must be given in exact form to demonstrate equality B0M1A1 if *BA* or *AB* not explicit B0M1A1 if *BA* or *AB* incorrect, as long as of form $\pm 6\mathbf{i} \pm \mathbf{j} \pm \mathbf{k}$ If using cosine rule, then B1 – state correct cosine rule M1 – attempt evaluation A1 – conclude convincingly, including use of surd value for cos 42°
7	(i)	$f(0.7) = 0.0648 > 0$ $f(0.8) = -0.103 < 0$ Sign change hence root	M1 A1 [2]	Evaluate at both 0.7 and 0.8 Conclude by referring to sign change oe CWO
	(ii)	Graph of $y = x$ and $y = \cos x$	B1 B1 [2]	Sketch both graphs… … in correct proportion to each other and intercepts correct
	(iii)	$\frac{dy}{dx} = -\sin x$ since $0 < x < \pi/2$ the magnitude of $-\sin x$ is less than 1 therefore the iteration converges or e.g. *x*: 0.7, 0.8; d*y*/d*x*: –0.64…, –0.71… magnitude of gradient in the region is less than 1 therefore the iteration converges	B1 M1 A1 [3]	State correct derivative Consider magnitude of gradient, either in general terms or at specific value(s) Allow use of $^{\pi}/_{4}$ as a specific value Conclude using $\lvert F'(x)\rvert < 1$ Allow $-1 < F'(x) < 0$ A0 for $\lvert F'(x)\rvert \leqslant 1$ A0 for $\lvert F'(x)\rvert < 1$ from $0 \leqslant x \leqslant {}^{\pi}/_{2}$, unless end point clearly dealt with

Table in 7(iii):

x	0.7	0.8
d*y*/d*x*	–0.64…	–0.71…

(iv)		M1 A1 [2]	First two segments At least 5 segments Allow **(ii)** and **(iv)** on the same graph
(v)	cos (0.73905) – 0.73905 = +5.879…x10^{-5} cos (0.73915) – 0.73915 = –1.085…x10^{-4} By the sign change rule α lies in that interval and therefore rounds to 0.7391 to 4 dp.	M1 A1 [2]	Evaluate at both 0.73905 and 0.73915 (or values closer to the root) Conclude by referring to sign change CWO
8	$4^2 = r^2 + r^2 - 2r^2 \cos\theta$ $r^2(1-\cos\theta) = 8$ Arc $PQ = r\theta = \theta\sqrt{\dfrac{8}{1-\cos\theta}}$	B1 M1 A1 M1 A1 [5]	State $4^2 = r^2 + r^2 - 2r^2 \cos\theta$ Attempt to make r, or r^2, the subject Obtain a correct expression for r, or r^2 Attempt to eliminate r from $s = r\theta$ Obtain correct arc length, aef For expressions that involve f(½θ): B1 – correct expression involving r and ½θ (e.g. right-angled trig, Sine Rule etc.) M1 – attempt to eliminate r from $s = r\theta$ M1 – attempt to use a correct identity to link f(½θ) and cosθ A1 – obtain correct identity A1 – obtain correct arc length, aef

9	(i)		B1	$\sec x = \dfrac{1}{\cos x}$ oe seen anywhere
		$\dfrac{\sin x}{1+\sin x} \equiv \dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}$	M1	Multiply top and bottom by $1 - \sin x$
		$\equiv \dfrac{\sin x - \sin^2 x}{1-\sin^2 x}$	A1	Obtain correct unsimplified expression
		$\equiv \dfrac{\sin x - 1 + \cos^2 x}{\cos^2 x}$	M1	Write denominator as $\cos^2 x$
		$\equiv \sec x \tan x - \sec^2 x + 1$	A1 [5]	Obtain correct simplified expression
		OR		
		$\sec x \tan x - \sec^2 x + 1 \equiv \dfrac{\sin x - 1 + \cos^2 x}{\cos^2 x}$		M1 – write with common denominator of $\cos^2 x$
		$\equiv \dfrac{\sin x - \sin^2 x}{1-\sin^2 x}$		M1 – attempt expression in terms of sin$x$ only
		$\equiv \dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)} \equiv \dfrac{\sin x}{1+\sin x}$		A1 – obtain correct unsimplified expression A1 – obtain correct simplified expression
	(ii)	$\displaystyle\int_0^{\frac{1}{4}\pi} \dfrac{\sin x}{1+\sin x}\,dx = \int_0^{\frac{1}{4}\pi} \sec x \tan x - \sec^2 x + 1\,dx$	M1	Attempt integration of given expression (at least two terms)
			A1	Obtain at least two correct terms (allow if third term not yet integrated)
		$= [\sec x - \tan x + x]_0^{\frac{1}{4}\pi}$	A1	Obtain fully correct integral
		$= (\sqrt{2} - 1 + \tfrac{1}{4}\pi) - (1 - 0 + 0)$	M1	Attempt correct use of limits (correct order and subtraction) in their integration attempt
			B1	State or imply sec $\tfrac{1}{4}\pi = \sqrt{2}$
		$= \tfrac{1}{4}\pi + \sqrt{2} - 2$ **AG**	A1	Obtain given answer convincingly
			[6]	Allow non 'hence' methods

10 (i)	$u=\frac{1}{x}$ and $\frac{du}{dx}=-\frac{1}{x^2}$	M1*	Attempt to link du and dx, to obtain kx^{-2}
	So $\int\frac{\sin\left(\frac{1}{x}\right)}{x^2}dx=\int -\sin u\,du$	A1	Correct integrand in terms of u
	$=\cos u+c=\cos\left(\frac{1}{x}\right)+c$	M1d*	Attempt integration of their f(u) – of form $a$sin$u$
		A1 [4]	Correct integral in terms of x, including $+c$
(ii)		M1	Attempt correct use of limits in *their* integral from part **(i)** Allow M1 for muddles with fractions, such as $\cos(^1/_\pi)$
	$\int_{1/2\pi}^{1/\pi}\frac{\sin\left(\frac{1}{x}\right)}{x^2}dx=-2$	A1	Obtain –2 cwo
	$\int_{1/3\pi}^{1/2\pi}\frac{\sin\left(\frac{1}{x}\right)}{x^2}dx=2$	A1 [3]	Obtain 2 cwo
(iii)	$\int_{1/(n+1)\pi}^{1/n\pi}\frac{\sin\left(\frac{1}{x}\right)}{x^2}dx=\cos(n\pi)-\cos((n+1)\pi)$	B1	Correct general expression in terms of n (no FT on incorrect integral)
	$\cos(n\pi)$ = 1 if n is even and –1 if n is odd	M1	Consider values of $\cos(n\pi)$, or another relevant expression e.g. $-2\sin(n\pi+{}^{\pi}/_{2})$
	So the integral is either 1 + 1 = 2 if n even or –1 –1 = –2 if n odd	A1 [3]	Fully convincing argument (including relevant subtractions) from cwo

11 (a)	$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$	M1		Attempt $^1/_h\,[f(x+h)-f(x)]$
		A1		Obtain correct expression (allow unsimplified denominator of $x+h-x$)
	$=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\cdot\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}$	M1		Multiply top and bottom by $\sqrt{(x+h)}+\sqrt{x}$
	$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$	A1		Simplify expression as far as possible
	$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$	A1	[5]	Complete proof by considering $\lim h\to 0$
	OR			
	$f'(x)=\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$			M1 – attempt $^1/_h\,[f(x+h)-f(x)]$
	$=\lim_{h\to 0}\frac{\sqrt{x}\left(1+\frac{1}{2}\frac{h}{x}-\frac{1}{8}\frac{h^2}{x^2}+\ldots\right)-\sqrt{x}}{h}$			A1 – obtain correct expression (allow unsimplified denominator of $x+h-x$)
	$=\lim_{h\to 0}\frac{\sqrt{x}+\frac{1}{2}\frac{h}{\sqrt{x}}+h^2(\ldots)-\sqrt{x}}{h}$			M1 – attempt binomial expansion with h/x
	$=\lim_{h\to 0}\frac{1}{2}\frac{1}{\sqrt{x}}+h(\ldots)$			A1 – simplify expression as far as possible
	$=\frac{1}{2\sqrt{x}}$			M1 – complete proof by considering lim $h\to 0$ Could also go via $^{\delta x}/_{\delta y}$, from $x=y^2$
(b) (i)	$y-\sqrt{a}=\frac{1}{2\sqrt{a}}(x-a)$, $y-\sqrt{b}=\frac{1}{2\sqrt{b}}(x-b)$	M1		Attempt equations of both tangents
	$\frac{1}{2\sqrt{a}}(x-a)+\sqrt{a}=\frac{1}{2\sqrt{b}}(x-b)+\sqrt{b}$	A1		Obtain both correct equations
		M1		Eliminate one variable and attempt to solve – as far as a correct equation in which x appears only once Allow M1 if solving normals not tangents
	$x=\sqrt{ab}$ **AG**	A1		Obtain $x=\sqrt{ab}$, detail required
	$y=\frac{1}{2}\left(\sqrt{a}+\sqrt{b}\right)$ **AG**	A1	[5]	Obtain $y=\frac{1}{2}\left(\sqrt{a}+\sqrt{b}\right)$, detail required
(ii)	Any valid solution e.g. $a=4$ and $b=16$	M1		State a pair of values that give one integer coord
		A1	[2]	State a pair of values that give both integer coords