

CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9794 MATHEMATICS	
9794/02	Paper 2 (Pure Mathematics 2), maximum raw mark 80

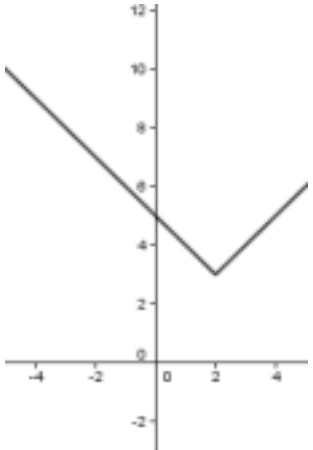
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

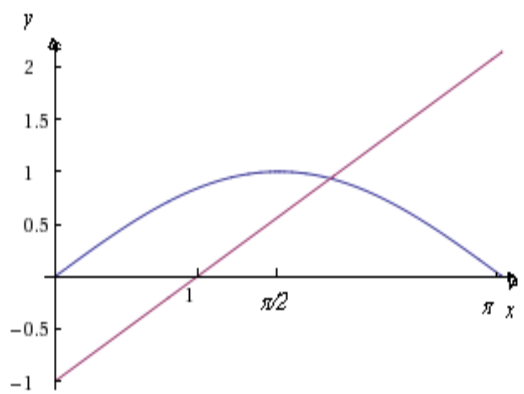
Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

Page 2	Mark Scheme	Syllabus	Paper
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1	$\frac{31}{6-\sqrt{5}} \times \frac{6+\sqrt{5}}{6+\sqrt{5}} = \frac{186+31\sqrt{5}}{31}$ $= 6+\sqrt{5}$	<p>M1 B1 A1</p> <p>[3]</p>	<p>Show intention to multiply top and bottom by $6+\sqrt{5}$</p> <p>Correct denominator; at least as far as $36-5$</p> <p>Show given answer correctly, including 31 seen as denominator before cancelling</p> <p>If showing that $(6+\sqrt{5})(6-\sqrt{5})=31$ then</p> <p>M1 – attempt to expand A1 – at least $36-5$ seen A1 – obtain 31</p>
2	$\int 6x^2 + 2dx = 2x^3 + 2x(+c)$ <p>$3 = 2 + 2 + c$ so $c = -1$ $y = 2x^3 + 2x - 1$</p>	<p>M1*</p> <p>A1 M1d* A1</p> <p>[4]</p>	<p>Attempt to integrate at least one term – increase in power by 1</p> <p>Obtain correct integral (allow no $+c$)</p> <p>Substitute (1, 3) to find c</p> <p>Correct equation, including $y =$</p>
3 (i)		<p>M1 A1 A1</p> <p>[3]</p>	<p>Sketch V-shape graph, vertex in any quadrant</p> <p>Vertex at (2, 3), y-intercept at 5</p> <p>Fully correct graph for at least $-5 \leq x \leq 5$</p>
	(ii) Two x -values correspond to the same y -value	<p>B1</p> <p>[1]</p>	<p>Or give numerical example such as $f(1) = 4 = f(3)$</p> <p>Referring to just 'multiple' or 'many' x-values is B0</p> <p>Must be using correct $f(x)$, so not just $x-2$</p> <p>B0 if additional incorrect statement, such as 'many to one'</p>

4	$\pi \int_1^2 x^6 dx = \pi \left[\frac{x^7}{7} \right]_1^2$ $= \pi \left(\frac{128}{7} - \frac{1}{7} \right) = \frac{127}{7} \pi \quad (= 57.0 \text{ to 3 sf})$	<p>B1 M1 M1 A1</p> <p>[4]</p>	<p>State or imply correct formula for volume of revolution</p> <p>Attempt integration to obtain kx^7</p> <p>Attempt use of limits in any integration attempt (i.e. increase in power by 1) Must be correct order and subtraction (M0M1 is possible)</p> <p>Obtain $\frac{127\pi}{7}$, or 57.0 or better (allow $\pi \frac{127}{7}$)</p>
5	<p>(i) $f(1.5) = 0.497494\dots$ $f(2) = -0.090702\dots$</p> <p>(ii) e.g., starting with $x_0 = 1.5$ $x_1 = 1.9974\dots$ $x_2 = 1.9103\dots$ $x_3 = 1.9429\dots$ $x = 1.93$ to 2 dp</p> <p>(iii)</p>  <p>(iv) One point of intersection oe</p>	<p>M1 A1</p> <p>[2]</p> <p>B1 M1 A1</p> <p>[3]</p> <p>M1* A1</p> <p>[2]</p> <p>B1 d*</p> <p>[1]</p>	<p>Attempt evaluation of $f(1.5)$ and $f(2)$ – evaluation must be seen so $f(1.5) > 0$ is not sufficient</p> <p>Conclude correctly – refer to sign change oe Must have correct values for $f(1.5)$ and $f(2)$ Allow rounded or truncated values – 1sf, or better</p> <p>Correct first iterate – must start with $1.5 \leq x \leq 2$ $f(1.75) = 1.9839\dots$, $f(2) = 1.9092\dots$</p> <p>Correct iteration process (at least 3) Allow iteration in degrees (gives 1.0177...)</p> <p>Obtain 1.93 – must be 2dp exactly Must be clear conclusion for root so A0 for e.g. $x_6 = 1.93$</p> <p>Sketch attempt at sine graph, with period of 2π, and a positive linear graph, with negative y-intercept</p> <p>Both graphs fully correct for $[0, \pi]$, with some indication of scale on both axes and with $y = x - 1$ passing through $(\frac{\pi}{2}, \approx 0.6)$</p> <p>Allow 'they will not cross again' or equivalent</p>

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6	(i)	<p>$\frac{dT}{dt}$ is the rate of change of T</p> <p>$T - 20$ is difference between T and the temp of the room.</p> <p>k is the constant of proportionality negative since the temperature is decreasing.</p>	B1 B1	At least two correct points Fully correct explanation
	(ii)	$\int \frac{1}{T-20} dT = \int -k dt$ $\ln T-20 = -kt + c$ $\ln 60 = c$ $T - 20 = e^{-kt + \ln 60} = e^{-kt} e^{\ln 60} = 60e^{-kt}$ $T = 20 + 60e^{-kt}$	M1 A1 A1 M1 M1 A1	<p>Separate variables and attempt integration of both sides oe</p> <p>Correct $\ln(T - 20)$</p> <p>Correct $-kt$ (allow no $+c$)</p> <p>Attempt c using $T = 80, t = 0$ (clear detail required if rearrangement is done first e.g. $80 = 20 + A$)</p> <p>Could be using any function, following clear integration attempt</p> <p>Rearrange expression of form $\pm \ln T - 20 = \pm kt \pm c$ to given expression, including correct manipulation of logs and exponentials – allow if still in terms of c</p> <p>Must be sound algebra throughout</p> <p>Obtain $T = 20 + 60e^{-kt}$, detail required and no errors seen</p> <p>Must see $e^{-kt + \ln 60} = e^{-kt} e^{\ln 60} = 60e^{-kt}$ (oe in terms of c)</p>
	(iii)	$\ln 40 = -2k + \ln 60 \quad \text{OR} \quad 60 = 20 + 60e^{-2k}$ $2k = \ln \frac{3}{2} \quad e^{-2k} = \frac{2}{3}$ $k = \frac{1}{2} \ln \frac{3}{2} \quad k = -\frac{1}{2} \ln \frac{2}{3}$	M1 M1 A1	<p>Substitute $T = 60, t = 2$ into given expression, oe</p> <p>Attempt to find k, allow one slip but must be using correct order of operations</p> <p>Obtain $k = \frac{1}{2} \ln \frac{3}{2}$ oe, including 0.203</p>

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7	(i)	$x^3 = 27t^3$ $y = 1 + \frac{1}{27}x^3$ AG	M1	Attempt to eliminate t
	(ii)	$1 + \frac{1}{27}x^3 = x^2 + 4x - 19$ $x^3 - 27x^2 - 108x + 540 = 0$ $(x - 3)(x^2 - 24x - 180) = 0$ $(x - 30)(x + 6) = 0$ $x = 30$ or -6 points (30, 1001) and $(-6, -7)$ OR $1 + t^3 = 9t^2 + 12t - 19$ $t^3 - 9t^2 - 12t + 20 = 0$ $(t - 1)(t^2 - 8t - 20) = 0$ $(t - 1)(t - 10)(t + 2) = 0$ $t = 1, 10$ or -2 points (30, 1001) and $(-6, -7)$	M1 A1 [2]	Obtain given answer convincingly M1A0 for $y = 1 + \left(\frac{x}{3}\right)^3 = 1 + \frac{1}{27}x^3$
8		$f'(x) = \frac{2x(3x^2 - 1) - x^2(6x)}{(3x^2 - 1)^2}$ $= \frac{-2x}{(3x^2 - 1)^2}$ for $x > 0$, $-2x < 0$ and $(\dots)^2 > 0$ and $\frac{-ve}{+ve} < 0$ hence decreasing function	M1 A1 A1 M1 A1 M1d* A1 A1 [6]	Attempt use of quotient rule, or equivalent Correct unsimplified expression Correct simplified expression Identify that $f'(x) < 0$ is required; allow 'gradient' for $f'(x)$ Show convincingly that the denominator is always positive and the numerator is always negative for $x > 1$, and hence $f'(x) < 0$ Graphical solutions could get M1 for $f'(x) < 0$ is required, but need to show no stationary points to get any further credit
			A1 [5]	Obtain coordinates of both points

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<p>9</p>	$2y \frac{dy}{dx} = 4x^3 - 12x^2$ $4x^2(x-3) = 0$ <p>$x = 0$ or $x = 3$ $x = 0 \rightarrow y^2 = 36 \rightarrow y = \pm 6$ $x = 3 \rightarrow y^2 = 9 \rightarrow y = \pm 3$ hence equations are $y = 3, y = -3, y = 6, y = -6$</p>	<p>M1 A1 B1 M1 A1 M1 A1 A1</p>	<p>Differentiate implicitly to get at least LHS Obtain fully correct expression Use $\frac{dy}{dx} = 0$ Attempt to solve for x Obtain $x = 0, 3$, www Attempt to find y, must include square rooting Obtain at least two correct equations, www Obtain all four correct equations, and no others (A1 A0 if final eqns given as $y = \pm 3, y = \pm 6$)</p> <p>Misreading y for y^2 gets M0A0B1M1A1M1A1A0</p> <p>Using $y = \sqrt{x^4 - 4x^3 + 36}$ can get full marks</p> <p>[8]</p>
<p>10 (i)</p> <p>(ii)</p>	$\sin\left(2\theta + \frac{1}{2}\pi\right) = \sin 2\theta \cos \frac{1}{2}\pi + \sin \frac{1}{2}\pi \cos 2\theta$ <p>$\cos \frac{1}{2}\pi = 0, \sin \frac{1}{2}\pi = 1$ so</p> $\sin\left(2\theta + \frac{1}{2}\pi\right) = \cos 2\theta$ <p>(ii)</p> $\sin\left(2\theta + \frac{1}{2}\pi\right) = \sin 3\theta,$ <p>A: $2\theta + \frac{1}{2}\pi = 3\theta \Rightarrow \theta = \frac{1}{2}\pi$ B: $3\theta = \pi - \left(2\theta + \frac{1}{2}\pi\right)$</p> $\theta = \frac{1}{10}\pi$ $3\theta = \pi - \left(2\theta + \frac{1}{2}\pi\right) + 4\pi \Rightarrow \theta = \frac{9}{10}\pi$ $3\theta = \pi - \left(2\theta + \frac{1}{2}\pi\right) + 6\pi \Rightarrow \theta = \frac{13}{10}\pi$ $3\theta = \pi - \left(2\theta + \frac{1}{2}\pi\right) + 8\pi \Rightarrow \theta = \frac{17}{10}\pi$	<p>M1 A1 B1 M1 A1 A1 A1</p>	<p>Use correct expansion These values must be explicit or implied in method for A1 Obtain given answer convincingly Also allow arguments by linear transformations</p> <p>Obtain $\frac{1}{2}\pi$ Attempt second solution using symmetry of sin curve oe Obtain $\frac{1}{10}\pi$ Obtain $\frac{9}{10}\pi$ Obtain $\frac{13}{10}\pi$ Obtain $\frac{17}{10}\pi$</p> <p>Accept decimal equivalents for each root After B1M1A1 given, apply penalty of -1 against final three A marks for each additional incorrect root</p> <p>[6]</p>

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(iii)	$\sin(2\theta + \theta) = \sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ $= 2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta$ $= 2 \sin \theta - 2 \sin^3 \theta + \sin \theta - 2 \sin^3 \theta$ $= 3 \sin \theta - 4 \sin^3 \theta$ $\cos 2\theta - \sin 3\theta (= 0)$ $(1 - 2 \sin^2 \theta) - (3 \sin \theta - 4 \sin^3 \theta) (= 0)$ $4 \sin^3 \theta - 2 \sin^2 \theta - 3 \sin \theta + 1 (= 0)$ $x = 0.309, 1 \text{ or } -0.809 \text{ to } 3\text{sf}$	<p>M1*</p> <p>M1d*</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[6]</p>	<p>Expand using $\sin(2\theta + \theta)$ Or use De Moivre's theorem</p> <p>Attempt to get expression in terms of $\sin \theta$ only</p> <p>Obtain given answer convincingly</p> <p>Attempt to rearrange to comparable format</p> <p>Identify $x = \sin \theta$ (could be implied) and attempt to use solution(s) from part (ii)</p> <p>Obtain $x = 0.309, 1, -0.809$ (allow 2dp)</p> <p>Allow surd values of $\frac{1}{4}(-1 \pm \sqrt{5})$</p>
11 (i)	$RS = r\theta$ $RT = r \tan \theta$ $OT = r \sec \theta$ $ST = r \sec \theta - r$ $P = r \sec \theta - r + r\theta + r \tan \theta$ $A = \frac{1}{2} RT \times OR$ $= \frac{1}{2} r^2 (\tan \theta - \theta)$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>[7]</p>	<p>Correct RS</p> <p>Correct RT</p> <p>Attempt ST</p> <p>Fully correct expression for P (could be $\frac{1}{\cos \theta}$ for $\sec \theta$, but not $\sqrt{1 + \tan^2 \theta}$)</p> <p>Attempt area of triangle – must be valid attempt at RT</p> <p>State correct area of sector</p> <p>Correct expression for A</p>
(ii)	<p>Let $A = rP$,</p> $r^2 \sec \theta - r^2 + r^2 \theta + r^2 \tan \theta = \frac{1}{2} r^2 (\tan \theta - \theta)$ <p>so $2 \sec \theta - 2 + \tan \theta + 3\theta = 0$</p> <p>for $0 < \theta < \frac{1}{2}\pi$, $\sec \theta > 1$, so $2 \sec \theta - 2 > 0$.</p> <p>Since $\tan \theta > 0$ and $\theta > 0$ equality is impossible, so we have a contradiction.</p>	<p>M1</p> <p>M1</p> <p>B1</p> <p>A1</p> <p>[4]</p>	<p>Equate A with rP (allow use of \neq)</p> <p>Attempt to justify why no solutions – correct equation only</p> <p>State $\sec \theta > 1$, or equivalent</p> <p>Fully correct and convincing argument</p>