

MATHEMATICS

9794/02

Paper 2 Pure Mathematics 2

May/June 2014

2 hours

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF20)

* 4 8 8 7 6 7 1 7 8 6 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

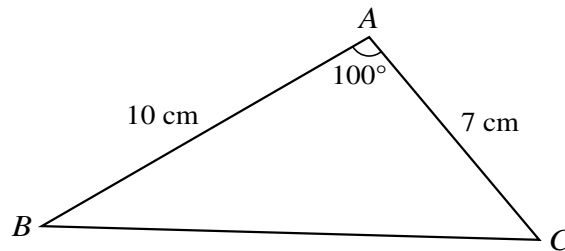
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

This document consists of **3** printed pages and **1** blank page.

- 1 The diagram shows the triangle ABC . $AB = 10$ cm, $AC = 7$ cm and angle $BAC = 100^\circ$.



- (i) Find the length BC . [2]
- (ii) Find the area of the triangle ABC . [2]
- 2 Let $f(x) = x^2 + kx + 4$, where k is a constant.
- (i) Find an expression for the discriminant of f in terms of k . [2]
- (ii) Hence find the range of values of k for which the equation $f(x) = 0$ has two distinct real roots. [3]
- 3 Given that $f(x) = x^3$, use differentiation from first principles to prove that $f'(x) = 3x^2$. [4]
- 4 The points A , B , C and D have coordinates $(2, -1, 0)$, $(3, 2, 5)$, $(4, 2, 3)$ and $(-1, a, b)$ respectively, where a and b are constants.
- (i) Find the angle ABC . [4]
- (ii) Given that the lines AB and CD are parallel, find the values of a and b . [3]
- 5 An arithmetic progression has first term 5 and common difference 7.
- (i) Find the value of the 10th term. [1]
- (ii) Find the sum of the first 15 terms. [2]
- The terms of the progression are given by x_1, x_2, x_3, \dots
- (iii) Evaluate $\sum_{n=1}^{15} (2x_n + 1)$. [3]
- 6 Given that the angle θ is acute and $\cos \theta = \frac{3}{4}$ find, without using a calculator, the exact value of $\sin 2\theta$ and of $\cot \theta$. [5]
- 7 (i) Express $z^4 + 3z^2 - 4$ in the form $(z^2 + a)(z^2 + b)$ where a and b are real constants to be found. [2]
- (ii) Hence draw an Argand diagram showing the points that represent the roots of the equation $z^4 + 3z^2 - 4 = 0$. [2]

8 Show that the graph of $y = x^2 - \ln x$ has only one stationary point and give the coordinates of that point in exact form. [6]

9 A new lake is stocked with fish. Let P_t be the population of fish in the lake after t years. Two models using recurrence relations are proposed for P_t , with $P_0 = 550$.

$$\text{Model 1 : } P_t = 2P_{t-1}e^{-0.001P_{t-1}}$$

$$\text{Model 2 : } P_t = \frac{1}{2}P_{t-1}\left(7 - \frac{1}{160}P_{t-1}\right)$$

(i) Evaluate the population predicted by each model when $t = 3$. [4]

(ii) Identify, with evidence, which one of the models predicts a stable population in the long term. [2]

(iii) Describe the long term behaviour of the population for the other model. [1]

10 Let $f(x) = x^4 - 4x^3 - 10x^2 + 28x - 15$.

(i) Show that $x = 1$ is a root of the equation $f(x) = 0$. [2]

(ii) Find the quotient and remainder when $f(x)$ is divided by $x - 5$. [4]

(iii) Factorise $f(x)$ fully and hence sketch the graph of $y = f(x)$. [5]

11 The cubic equation $x^3 - 2x^2 + 4x - 7 = 0$ has a single root α , close to 1.9, which can be found using an iteration of the form $x_{n+1} = F(x_n)$. Three possible functions that can be used for such an iteration are

$$F_1(x) = \frac{7}{4} + \frac{1}{2}x^2 - \frac{1}{4}x^3, \quad F_2(x) = \sqrt[3]{2x^2 - 4x + 7}, \quad F_3(x) = \frac{7 - 4x}{x^2 - 2x}.$$

(i) Differentiate each of these functions with respect to x . [5]

(ii) Without performing any iterations, and using $x = 1.9$, show that an iterative process based on only two of the given functions will converge.

Determine which one will do so more rapidly. [4]

The sequence of errors, e_n , is such that $e_{n+1} \approx F'(\alpha)e_n$.

(iii) Using the iteration from part (ii) with the most rapid convergence, estimate the number of iterations required to reduce the magnitude of the error from $|e_1|$ in the first term to less than $10^{-10}|e_1|$. [3]

12 A curve C is defined parametrically by

$$x = \cos t(1 - 2 \sin t), \quad y = \sin t(1 - 3 \sin t), \quad 0 \leq t < 2\pi.$$

(i) Show that C intersects the y -axis at exactly three points, and state the values of t and y at these points. [5]

(ii) Find the range of values of t for which C lies above the x -axis. [4]

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