

FURTHER MATHEMATICS (PRINCIPAL)

9795/02

Paper 2 Further Applications of Mathematics

May/June 2019

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

* 7 1 2 4 6 2 9 0 2 5 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use 10 m s^{-2} .

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 120.

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **7** printed pages and **1** blank page.

Section A: Probability (60 marks)

- 1 The random variable W has the distribution $Po(35)$.
- (a) Write down the values of the mean and the variance of W . [2]
- (b) Use a suitable approximation to find the smallest value of r for which $P(W \leq r) > 0.98$. [5]

- 2 Weeds can be assumed to occur randomly in a region of lawn.
- (a) State two further assumptions needed for the number of weeds in a fixed area of lawn to be well modelled by a Poisson distribution. [2]

Assume now that all these assumptions are valid, and that on average there are 5 weeds per square metre of lawn.

- (b) Find the probability that there are at least 7 weeds in a randomly chosen region of lawn of area 0.8 m^2 . [3]
- (c) Using tables, find the smallest area $A \text{ m}^2$ of lawn for which the probability that there are no more than 6 weeds is less than 0.92, giving your answer as an interval of the form $a_1 < A < a_2$. You should show the values of any relevant probabilities. [4]
- 3 A statistician measured how long it took boys and girls to complete a certain activity. In a random sample of 40 boys, the mean time was 13.40 minutes and an unbiased estimate of the population variance was 4.63 minutes^2 .
- (a) Explain why it is an advantage for a sample to be random. [1]
- (b) Calculate a 95% confidence interval for the mean time taken by all boys to complete the activity. Give the end-points correct to 2 decimal places. [3]
- (c) Explain whether it was necessary to use the Central Limit Theorem in your solution to part (b). [1]

The times y minutes taken to complete the activity by a random sample of 60 girls can be summarised by

$$\Sigma y = 1092, \quad \Sigma y^2 = 20\,324.4.$$

- (d) Calculate a 95% confidence interval for the difference in the mean time taken by boys and the mean time taken by girls to complete the activity. [6]
- (e) It was later found that many of the boys and girls in the samples were pairs of brothers or sisters. Explain whether this invalidates your calculation in part (d). [1]

- 4 A function $f(t)$ is defined as follows.

$$f(t) = \begin{cases} \frac{1}{4}\pi \sin(\frac{1}{2}\pi t) & 0 \leq t \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Show that $f(t)$ satisfies the requirements for a probability density function. [4]

A continuous random variable T has probability density function $f(t)$.

- (b) Write down the value of $E(T)$. [1]

- (c) Find the value of w which satisfies $P[E(T) - w < T < E(T) + w] = 0.9$. [3]

- (d) Find $E[\cos^2(\frac{1}{2}\pi T)]$. [4]

- 5 Two players play a game with a fair six-sided die with faces numbered 1 to 6. In each round of the game each player throws the die once. The game ends if either of the players throws a 6, unless both of them throw a 6 in the same round, in which case the game continues. The total number of throws of the die up to and including the last throw is denoted by X .

- (a) State the possible values that X can take. [1]

- (b) Show that $P(X = 2) = \frac{5}{18}$ and find $P(X = 4)$. [2]

- (c) Show that the probability generating function of X is $\frac{5t^2}{18 - 13t^2}$. [3]

- (d) The game is played twice. Use the probability generating function to find the probability that the total number of throws is 8. [4]

- 6 (a) Explain what is meant by an unbiased estimator of a population parameter θ . [2]

In the rest of this question, \bar{X} denotes the mean of a random sample of n observations of a random variable X .

- (b) Given that $E(X) = \mu$ and $\text{Var}(X) = E(X - \mu)^2 = \sigma^2$, write down the value of $E(\bar{X} - \mu)^2$ in terms of σ and n . [1]

- (c) By writing $[X - \mu]^2$ as $[(X - \bar{X}) + (\bar{X} - \mu)]^2$, and using $(A + B)^2 = A^2 + 2AB + B^2$, show that

$$E(X - \bar{X})^2 = \left(1 - \frac{1}{n}\right)\sigma^2.$$

[You may assume without proof that $E[(X - \bar{X})(\bar{X} - \mu)] = 0$.] [6]

- (d) Hence write down an unbiased estimator of σ^2 . [1]

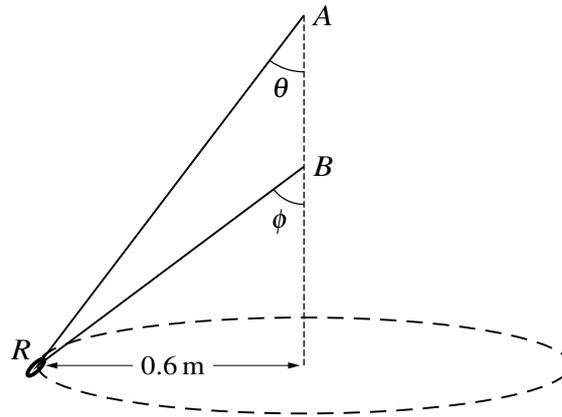
Section B: Mechanics (60 marks)

7 A car of mass 700 kg travels along a horizontal road. Its speed increases from 15 m s^{-1} to 25 m s^{-1} . The work done by the engine of the car is W_e joules and the work done against friction is W_f joules.

(a) Find an equation connecting W_e and W_f . [2]

(b) The total distance travelled is 200 m. The frictional force can be modelled by a constant force of 150 N for the first 100 m and a constant force of 250 N for the second 100 m. Find the value of W_e . [2]

8

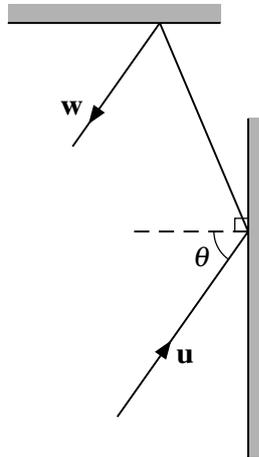


A small smooth ring R of mass 0.04 kg is threaded on a light inextensible string AB . The ends A and B of the string are fixed to two points, with A vertically above B . The ring moves at constant speed in a horizontal circle of radius 0.6 m with the string taut. The centre of the circle is vertically below A and B . The angles between RA and the vertical and RB and the vertical are denoted by θ and ϕ respectively, and $\theta + \phi = \frac{1}{2}\pi$ (see diagram).

(a) Find the angular velocity of the ring. [7]

(b) Given that $\cos \theta = 0.8$, find the tension in the string. [3]

- 9 A snooker ball of mass 0.2 kg moving on a smooth horizontal table hits a smooth cushion. Immediately before the impact, the velocity \mathbf{u} of the ball makes an angle of θ with the normal to the cushion. After the first impact, the ball hits another smooth cushion at right angles to the first (see diagram). In both collisions the coefficient of restitution between the ball and the cushion is 0.6. The velocity of the ball after the second impact is denoted by \mathbf{w} .



- (a) Show that $\mathbf{w} = -k\mathbf{u}$ and state the value of k . [5]
- (b) It is given that the magnitude of \mathbf{u} is 0.7 m s^{-1} . Find the magnitude and direction of the total impulse on the ball as a result of the two impacts. [4]

- 10 A uniform rectangular lamina $ABCD$ of weight 12 N has sides $AB = 2\text{ m}$ and $AD = 2\sqrt{3}\text{ m}$.

(a)

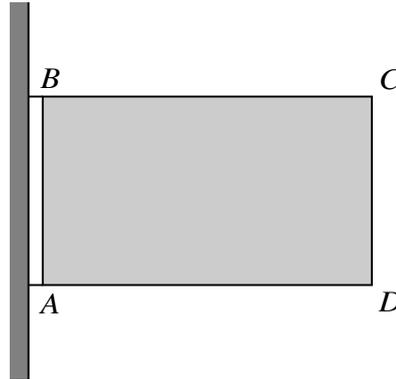


Fig. 1

The points A and B of the lamina are fixed to a vertical wall and the lamina rests in equilibrium in a vertical plane perpendicular to the wall (see Fig. 1). Find the horizontal components of the forces on the lamina at A and B . [3]

(b)

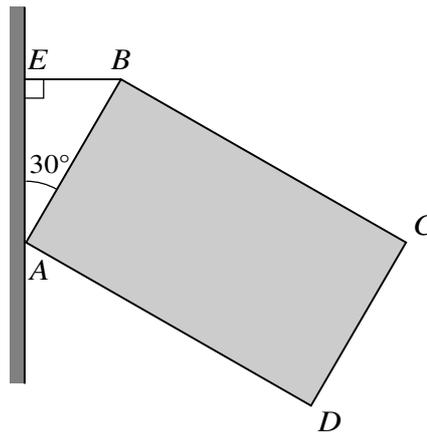


Fig. 2

The lamina is now held in a vertical plane perpendicular to the wall by a light inextensible string attached to B and to the wall at E . AB makes an angle of 30° with the upward vertical, the string is horizontal, and the lamina is in equilibrium with A in contact with the wall (see Fig. 2). The coefficient of friction between the wall and the lamina is μ . Find the tension in the string and the least possible value of μ . [7]

11 A hollow sphere of internal radius 0.4 m has a smooth inner surface. A particle is placed at the lowest point of the inner surface of the sphere and is then given an initial horizontal velocity of magnitude $u \text{ m s}^{-1}$.

- (a) Given that the particle moves in a complete vertical circle on the inner surface of the sphere, find the smallest possible value of u . [5]

It is given instead that the particle loses contact with the sphere at the point where the angle θ between the radius and the upward vertical is $\cos^{-1}\left(\frac{3}{5}\right)$.

- (b) Find the radial and tangential components of the acceleration of the particle at the point where it loses contact with the sphere. [3]

- (c) Hence find the values of $\dot{\theta}$ and $\ddot{\theta}$ at this point. [3]

12 The acceleration of a particle moving with speed v at time t is given by $\frac{k(u^2 - v^2)}{2u}$, where k and u are constants.

- (a) Write down a differential equation satisfied by v , and find the general solution for v in terms of t , u and k . [7]

- (b) It is given that $k = 0.2$ and that $v = 0$ when $t = 0$.

- (i) Find an expression for v in terms of t and u . [2]

- (ii) State with a reason what happens to v for large t . [2]

- (c) It is given instead that $k = -0.2$ and that $v = 2u$ when $t = 0$.

- (i) Find an expression for v in terms of t and u . [2]

- (ii) Explain why the solution to the differential equation cannot be used to describe the motion when t is larger than a certain value T to be found. [3]

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