
FURTHER MATHEMATICS

9795/02

Paper 2 Further Applications of Mathematics

May/June 2016

MARK SCHEME

Maximum Mark: 120

Published

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1	(i)	$75 \pm 1.96 \sqrt{\frac{40^2}{500} \times \frac{500}{499}}$ $= (71.5, 78.5)$	M1 B1 A1 A1 [4]	$75 \pm zs$, s involving 500 $z = 1.96$, allow from no 500 Variance correct Both limits correct to 3sf	<div style="border: 1px solid black; padding: 5px; width: fit-content;"> Condone omission of $\frac{500}{499}$ </div>
	(ii)	No, as the Central Limit Theorem applies OR as n is large	B1 [1]	“No” and mention CLT or large sample size; focus on different distributions; no irrelevancies	
2	(i)	$N(120, \dots)$ $\sigma^2 = 0.8^2 \times 1200 [= 768]$ $1 - \Phi((140 - 120)/\sqrt{768})$ $= 0.235(3)$	M1 M1 A1 A1 [4]	Normal, mean 120 or 1.20 Allow 0.8×1200 etc Both parameters correct Answer, in range [0.235, 0.236] OR: $P(\geq 175)$ from $N(150, 200)$ M1 $(175 - 150)/\sqrt{1200}$ A1 0.235(3) A2	
	(ii)	$B_1 + B_2 + B_3 + B_4 - S_1 - S_2 - S_3$ $\sim N(60, \dots)$ Variance $4 \times 1200 + 3 \times 1500 = 9300$ $\Phi\left(\frac{0 - 60}{\sqrt{9300}}\right) = \Phi(-0.622) = 0.267$	M1 M1 A1 A1 [4]	Consider $\pm(B_1 + B_2 + B_3 + B_4 - S_1 - S_2 - S_3)$ or $4B - 3S$ Normal, mean 60 Correct variance Answer, a.r.t. 0.267 [0.2699] [NB: $\Phi(-60/\sqrt{33700}) = 0.3700$ is 2/4]	
3	(i)	$\sum t^r P(r) = \sum_{r=0}^n t^r {}^n C_r p^r (1-p)^{n-r}$ $= \sum_{r=0}^n (pt)^r {}^n C_r (1-p)^{n-r}$ $= (1-p + pt)^n \quad \text{AG}$	M1 A1 A1 [3]	Use $\sum t^r P(R=r)$ and binomial probabilities Indicate correct final term Collect p^r and t^r and correctly obtain given expression OR $(1-p + pt)^n$, M1A1; answer, A1	
	(ii)	$\left(\frac{3}{4} + \frac{1}{4}t\right)^8 \left(\frac{1}{4} + \frac{3}{4}t\right)^8$ $= (3+t)^8 (1+3t)^8 / 4^{16}$ $= \left(\frac{3}{16} + \frac{1}{16}t(10+3t)\right)^8 \quad \text{AG}$ $= \left(\frac{3}{16}\right)^8 \left(1 + \left[\frac{10}{3} + t\right]t\right)^8$ $t \text{ term: } \left(\frac{3}{16}\right)^8 \left[8 \times \frac{10}{3}\right] = 4.07 \times 10^{-5}$	M1 A1 M1M1 A1 [5]	Substitute and multiply Correctly obtain given answer Select t term; method for expansion formula Answer OR: attempt to find $G'(0)$ M1 $8\left(\frac{3}{16}\right)^8 \left(\frac{10}{3} + 2t\right) \left(1 + \left[\frac{10}{3} + t\right]t\right)^7$ A1 $= 4.07 \times 10^{-5}$ A1	

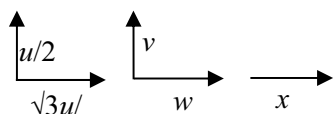
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4	(i)	Number of goals scored by home team is independent of number of goals scored by away team	B1 [1]	Not just <i>goals</i> independent. Extras, including conditions already implied by given Poisson distributions: B0
	(ii) (a)	$e^{-4.2n} \left(1 + 4.2n + \frac{(4.2n)^2}{2!} + \frac{(4.2n)^3}{3!} \right)$	M1 A1 A1 [3]	Po(4.2n) implied Correct ± 1 term Fully correct expression, aef. SR Po(4.2): Fully correct formula B1
	(b)	$e^{-2.4} e^{-1.8} (1 + 2.4 \times 1.8)$ $= \mathbf{0.0798}$	M1 A1 A1 [3]	Individual Poisson distributions multiplied Correct expression [= 0.0150 + 0.0647] Answer, a.r.t. 0.080 [0.07977]
5	(i)	n large p close to $\frac{1}{2}$	B1 B1 [2]	Or $np > 5$ $nq > 5$ [<i>not</i> $npq > 5$]
	(ii)	$\frac{24.5 - \mu}{\sigma} = \Phi^{-1}(0.8282) = 0.947$ $\frac{27.5 - \mu}{\sigma} = \Phi^{-1}(0.9697) = 1.759$ $\mu = \mathbf{21}, \sigma = \mathbf{3.69}$	M1 A1 B1 M1 A1 [5]	One standardised, = Φ^{-1} , allow σ^2 , cc, 1- errors LHS of both equations correct including signs and cc Both z -values correct to 3 sf, ± 1 in third dp Solve to find both μ and σ μ , a.r.t. 21.0; σ , in range [3.69, 3.70]
	(iii)	$q = npq/np = 21/3.694^2$ [= 0.65] $p = \mathbf{0.35}, n = \mathbf{60}$	M1 A1ft A1 [3]	Correct method of solution for n, p or q , allow \sqrt{npq} $npq = \sigma^2$ [not σ], ft on their npq [13.65] p , a.r.t. 0.350 and $n = 60$ [integer] <i>only</i> [not 60.0]
6	(i)	$\int_0^\infty 4xe^{-2x} e^{tx} dx = \int_0^\infty 4xe^{-(2-t)x} dx$ $= \left[\frac{4xe^{-(2-t)x}}{(t-2)} \right]_0^\infty + \int_0^\infty \frac{4e^{-(2-t)x}}{2-t} dx$ $\left[-\frac{4e^{-(2-t)x}}{(2-t)^2} \right]_0^\infty = \frac{4}{(2-t)^2}$	M1 A1 M1 A1 A1 [5]	Attempt $\int e^{tx} f(x) dx$, limits somewhere Combine into single e term Use parts, right way round Correct indefinite integral Correct final answer, cwo, allow $(t-2)^2$ but must use integral that visibly converges, or otherwise indicate the issue
	(ii)	$t < 2$	B1 [1]	
	(iii)	$\left[\frac{4}{(2-t)^2} \right]^3 = \frac{64}{(2-t)^6}$ $= (1 - \frac{1}{2}t)^{-6} = 1 + 3t + \frac{21}{4}t^2 + \dots$ $E(Y) = \mathbf{3}$ $E(Y^2)/2 = 21/4$ so $E(Y^2) = 10.5$ $\text{Var}(Y) = 10.5 - 3^2 = \mathbf{1.5}$	M1 A1 M1 A1 A1 [6]	$[M_X(t)]^3$ [Not cubed: M0A0 M1A0 M1A0] Series expansion <i>or</i> differentiate once $M'(t) = \frac{384}{(2-t)^7}, M''(t) = \frac{2688}{(2-t)^8}$ $E(Y) = 3$ correctly obtained or implied $2 \times$ coeff of t^2 <i>or</i> $M''(0) - [M'(0)]^2$ $\text{Var}(Y) = 1.5$ or exact equivalent, cwo

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7	(i)	$\int_0^k x \frac{3x^2}{k^3} dx = \frac{3}{4}k$ $E\left(\frac{4}{3}X\right) = k, \text{ so } \frac{4}{3}X \text{ unbiased AG}$	M1 A1 A1 [3]	Attempt $\int xf(x)$, correct limits $\frac{3}{4}k$, ae exact f Must state “unbiased”
	(ii)	$P(X \leq x) = \int_0^x \frac{3x^2}{k^3} dx = \left(\frac{x^3}{k^3}\right)$ $P(M \leq m) = \left(\frac{x^3}{k^3}\right)^3 = \frac{x^9}{k^9}$ $f_M(x) = 9 \frac{x^8}{k^9} \quad \text{AG}$	B1 M1 M1 A1 [4]	Needs convincing derivation [$F_X(x)$] ³ Differentiate Full derivation of AG. Ignore other ranges
	(iii)	$\int_0^k x 9 \frac{x^8}{k^9} dx = \frac{9}{10}k$ <p>Hence $E_2 = \frac{10}{9}M$</p>	M1 A1 A1ft [3]	Attempt $\int xf_M(x)$, ignore limits Correct $E(M)$ If $E(M) = kc$, allow M/c
8		<p>PE lost = $0.4g \times 3 \sin 20^\circ$ [4.104] Initial KE = $\frac{1}{2} \times 0.4 \times 0.5^2$ [0.05] Final KE = $\frac{1}{2} \times 0.4 \times 2.5^2$ [1.25] Difference = Work done by friction $2.9045 = 3F$ Therefore $F = \mathbf{0.968 N}$</p>	M1 M1 M1 M1 A1 [5]	mgh attempted, with trig Both KEs attempted Work/Energy principle used, no extra/missing terms Use $\Delta E = F \times s$ Answer, a.r.t. 0.968 [SC: Energy not used: answer B2]
9	(i)	$R(\uparrow_Q): T - 1.5g = 0 \quad [T = 15]$ $R(\uparrow_P): T \cos \theta - 1.2g = 0$ $[15 \cos \theta = 12]$ $\Rightarrow \cos \theta = 0.8$ <p>Distance below = $0.12/\tan \theta$ = $\mathbf{0.16m}$</p>	M1 M1 A1 A1 [4]	Resolve vertically for 1.5 kg mass, can be implied e.g. by $T = 15$ Resolve vertically for 1.2 kg mass Value of $\cos \theta$ [$\theta = 36.9^\circ$] Correct value of h
	(ii)	$T = 1.5g$ $R(\rightarrow_P): T \sin \theta = 1.2r\omega^2 \quad [= 9]$ $\Rightarrow 1.5g \sin \theta = 1.2 \times 0.12 \times \omega^2$ $[a = 7.5]$ $\Rightarrow \omega = \sqrt{62.5} = \mathbf{7.91 \text{ rad s}^{-1}}$	B1 M1 A1 A1 [4]	Value of T used [= 15] Resolve horiz for P and use $r\omega^2$ or v^2/r Correct equation, and $v = r\omega$ if v used Answer, in range [7.9, 7.91] or $\frac{5}{2}\sqrt{10}$

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10	(i)	$N = 80$ $M(X): F \times 5 \sin \theta = 80 \times 2.5 \cos \theta$ $F \leq 0.4N$ $\tan \theta \geq 1.25$ $\theta_{\min} = 51.3^\circ$ or $51.4^\circ, 0.896$	B1 M1 M1 M1 A1 [5]	Normal force at ground (can be implied) Moments about any point, needs both cos and sin $A: R \times 5 \sin \theta = 80 \times 2.5 \cos \theta$ $[R = F = 0.4N = 32]$ $M: R \times 2.5 \sin \theta + F \times 2.5 \cos \theta = N \times 2.5 \cos \theta$ Use $F \leq \mu N$ or $F = \mu N$ Solve equations to obtain $\tan \theta$ Correct answer, in range $[51.3, 51.4]$ or a.r.t. 0.896
	(ii)	$F \times 5 \sin \theta = (80 \times 2.5 + 750d) \cos \theta$ Use 60° and μ to obtain $d_{\max} = 3.57$	M1* depM1 A1 [3]	Moments equation with variable (d) [$F = 332$] $[332 \times 5 \times 0.5 = 100\sqrt{3} + 337\sqrt{3}d]$ Answer, a.r.t. 3.57 or 3.56
11	(i)	Driving force = $32000/v$ $800 \frac{dv}{dt} = \frac{32000}{v} - 20v$ $\frac{dv}{dt} = \frac{1600 - v^2}{40v}$ AG	M1 A1 [2]	Use P/v and differential equation including dv/dt Correctly obtain AG, need to use 800 convincingly
	(ii)	$\int \frac{40v}{1600 - v^2} dv = \int dt$ $c - 20 \ln(1600 - v^2) = t$ $c = 20 \ln 1600$ [147.56] $t = 20 \ln \left(\frac{1600}{1600 - v^2} \right)$ $v = 40\sqrt{1 - e^{-t/20}}$ Tends to 40	M1 A1 A1 M1 A1 B1 [6]	Separate variables and attempt to integrate Correct indefinite integral, aef Correct value of c Make v subject, using e , allow v^2 Correct expression for v , aef Conclusion, cwo but can get from implicit formula
12	(i)	 $\text{Mom}^m (\rightarrow): mu \cos 30 = mw + mx$ $\text{Rest}^n: x - w = 0.9\sqrt{3}u/2$ Solve: $w = 0.0433u$ $\text{Mom}^m (\uparrow): mu \sin 30 = vm$ so $v = 0.5u$ \Rightarrow direction is $\tan^{-1}(0.5/0.0433)$ $= 85.05^\circ$ to x -axis	M1 M1 A1 B1 A1 [5]	$[\frac{\sqrt{3}}{2}u = w + x]$ C of M equation, needn't have m , ignore signs, needs cos and sin Restitution equation, ignore signs of LHS Correctly obtain $w = \text{a.r.t. } 0.0433u$ [= $\sqrt{3}u/40$] Obtain, state, or use $v = u/2$
	(ii)	$u \cos 30 = w + x, ue \cos 30 = x - w$ $\Rightarrow 2w = u \cos 30(1 - e)$ but $e \leq 1$ so w cannot be negative	M1 M1 A1 [3]	One general equation Second equation, and use $e \leq 1$ Correctly deduce given conclusion

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13 (i)	Solution 1: V, H		
	$y = 30t \sin 50^\circ - 5t^2; x = 30t \cos 50^\circ$ $[y = 26.2] \quad x = 48.2$ Height of slope = $x \tan 10^\circ = 8.5$ Difference = 17.7 Solution 2: \parallel, \perp $y = 30t \sin 40^\circ - 5t^2 \cos 10^\circ$ $= 17.43$ Height above field = $y \div \cos 10^\circ$ $= 17.7$	M1A1 A1 M1 A1 [5]	Both equations attempted; both correct Correct value of x SC: If M0, $y = 26.2$ gets B1 Find height Correct answer [17.703]
13 (ii)	Solution 1: V, H		
	$30t \sin 50^\circ - 5t^2 = 30t \cos 50^\circ \tan 10^\circ$ $t = 3.916$ $PX = 30t \cos 50^\circ \div \cos 10^\circ$ $X = 76.7$ Solution 2: \parallel, \perp $Y = 30t \sin 40^\circ - 5t^2 \cos 10^\circ$ $Y = 0$ at $t = 3.916$ $X = 30t \cos 40^\circ - 5t^2 \sin 10^\circ$ $PX = 76.7$ Solution 3: traj $Y = X \tan 50^\circ - \frac{10X^2 \sec^2 50^\circ}{2 \times 30^2}$ $= x \tan 10^\circ$ $PX = 180(\tan 50^\circ - \tan 10^\circ) \cos^2 50^\circ =$ 76.7	M1 A1 M1 A1 [4]	Put $y = x \tan 10^\circ$ and solve Correct value of t , can be implied Calculate $x \div \cos 10^\circ$ or $y \div \sin 10^\circ$ X , a.r.t. 76.7 [76.684]
		M1 A1 A1 A1	Y equation, allow sign/trig errors (3 terms) Correct value of t , can be implied X equation, allow sign/trig errors (3 terms) X , a.r.t. 76.7 [76.684]
	M1 A1	Use trajectory equation All correct	
	M1 A1	Solve for X X , a.r.t. 76.7 [76.684]	

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14	(i)	$T - mg = 0 \Rightarrow \frac{3}{0.5} \times e = 0.3g$ $\Rightarrow e = 0.5$	M1	Use N2 for equilibrium
			A1	Equilibrium extension 0.5, a.r.t. 0.500
			[2]	
	(ii)	<p>GPE lost = EPE gained:</p> $3(0.5 + x) = \frac{1}{2} \frac{3}{0.5} x^2 \quad [= 3x^2]$ <p>Solve to get $x = \frac{1 + \sqrt{3}}{2} = 1.37$</p>	M1*	Use cons of energy, need variable on both sides, e.g. $3(1 + h) = 3(h + \frac{1}{2})^2$ or $3h = 3(h - \frac{1}{2})^2$
		depM1	Obtain and solve quadratic equation	
		A1	Correct equation, add/subtract 0.5 etc if necessary	
		A1	Solve to get a.r.t. 1.37 <i>only</i> , allow surds	
		[4]		
	(iii)	$0.3\ddot{x} = 0.3g - \frac{3}{0.5}(x + e)$ $\ddot{x} = -20x$	M1	Use N2 including extension
		A1	All correct, check signs	
		B1	Obtain correct value of ω^2 , no wrong working	
		[3]		
	(iv)	$x = \frac{\sqrt{3}}{2} \cos \sqrt{20}t$ $x = 0.5 \text{ at } \omega t = 0.9553 \text{ or } 5.3279$	M1	Use $x = a \cos \omega t$ or $a \sin \omega t$, allow $a = 1$
		A1ft	a correct ft (= their (ii) – their (i)), ω from (iii)	
		M1	Equate to $(\pm)e$ and use correct trig method e.g. $\frac{2}{\sqrt{20}} \cos^{-1} \left(-\frac{1}{\sqrt{3}} \right)$ or $\frac{1}{\sqrt{20}} \left(\frac{\pi}{\omega} + 2 \sin^{-1} \left(\frac{1}{\sqrt{3}} \right) \right)$	
		A1	One correct value of t	
		A1	Correct final answer [0.9777]	
		[5]		