Cambridge Assessment International Education

Cambridge Pre-U

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **October/November 2020**

3 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

INSTRUCTIONS
- Answer **all** questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

INFORMATION
- The total mark for this paper is 120.
- The number of marks for each question or part question is shown in brackets [ ].

JC20 11_9795_01/FP

1 Using standard summation results, prove that $\sum_{r=1}^{n}(4r^3 - 6r^2 + 4r - 1) = n^4$. [4]

2 The parabola $y = px^2 + qx + r$ passes through the points $(-1, -1)$, $(9, 53)$ and $(-11, 45)$.

(a) **(i)** Write down a system of three equations in p, q and r. [2]

(ii) Formulate this system as a matrix equation in the form $\mathbf{Cx} = \mathbf{a}$, where $\mathbf{C}$ is a 3×3 matrix, $\mathbf{x}$ is an unknown column vector and $\mathbf{a}$ is a constant vector. [1]

(b) Using any suitable method, determine the values of p, q and r. [4]

3 **(a)** **(i)** Write down the equations of the asymptotes of the curve $y = \dfrac{x-1}{x-4}$. [2]

(ii) Sketch this curve, showing all significant features. [4]

(b) Determine the equation of the oblique asymptote of the curve $y = \dfrac{(x-1)^2}{x-4}$. [2]

4 A curve has polar equation $r = 3 + \sqrt{2}\sin\theta$, for $\frac{1}{4}\pi \leqslant \theta \leqslant \frac{3}{4}\pi$. Find, in its simplest exact form, the area of the region enclosed by the curve and the lines $\theta = \frac{1}{4}\pi$ and $\theta = \frac{3}{4}\pi$. [6]

5 The equation $2x^3 + 3x^2 - 5x - 12 = 0$ has roots α, β and γ.

(a) State the value of $\alpha\beta\gamma$. [1]

A second cubic equation, with integer coefficients, has roots $\alpha + \dfrac{12}{\beta\gamma}$, $\beta + \dfrac{12}{\gamma\alpha}$ and $\gamma + \dfrac{12}{\alpha\beta}$.

(b) **(i)** Show that these new roots can be written as 3α, 3β and 3γ respectively. [2]

(ii) Find the second cubic equation. [3]

6 **(a)** Given the matrix $\mathbf{X} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$, calculate $\mathbf{X}^2$, $\mathbf{X}^3$ and $\mathbf{X}^4$. [3]

(b) Conjecture an expression for $\mathbf{X}^n$ for positive integers n and prove the result by induction. [4]

(c) Is the result still true when $n = -1$? Justify your answer. [3]

7 **(a)** **(i)** Express the complex number $\omega = 1 + \mathrm{i}\sqrt{3}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. [2]

(ii) Hence show that ω^7 is an integer multiple of ω. [3]

(b) Solve the equation $z^7 = 64 - 64\mathrm{i}\sqrt{3}$. Give each answer in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $0 < \theta < 2\pi$. [5]

8 A non-abelian group G, with identity element e, contains an element a of order 4 and an element b such that $a^3b = ba$.

(a) State, with justification, whether G is a cyclic group. [1]

(b) Show, in any order, that

- $b = aba$,
- $b = a^2ba^2$,
- $ba^3 = ab$.

Justify fully each step of your working. [7]

9 The function f is defined for $-1 \leqslant x \leqslant 1$ by $\mathrm{f}(x) = \cos^{-1} x$.

(a) **(i)** Sketch the graph of $y = \mathrm{f}(x)$. [1]

(ii) Given that $y = \cos^{-1} x$, prove that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\dfrac{1}{\sqrt{1-x^2}}$. [4]

(b) Determine $\displaystyle\int \cos^{-1} x \,\mathrm{d}x$. [5]

10 **(a)** Use the vector product to find the area of triangle ABC with vertices $A\,(1,\ 2,\ 3)$, $B\,(5,\ 1,\ -3)$ and $C\,(2,\ 3,\ -1)$. [4]

(b) **(i)** Calculate the volume of tetrahedron $OABC$, where O is the origin. [3]

(ii) Deduce the shortest distance from O to the plane ABC. [2]

(c) Determine the shortest distance between the line through O and A and the line through B and C. Give your answer in an exact surd form. [5]

11 The curve C has equation $y = \frac{2}{3}x^{\frac{3}{2}}$ for $0 \leqslant x \leqslant 15$.

(a) The length of C is denoted by L. Showing full working, determine the value of L. [4]

(b) The area of the surface generated when C is rotated once about the x-axis is denoted by A.

(i) Show that $A = \frac{4}{3}\pi \displaystyle\int_0^{15} x\sqrt{\left(x+\tfrac{1}{2}\right)^2 - \tfrac{1}{4}}\,\mathrm{d}x$. [3]

(ii) Use a suitable substitution to show that the exact value of A is

$$406\pi\sqrt{15} + \tfrac{1}{12}\pi\ln\left(31 + 8\sqrt{15}\right).$$ [8]

12 It is given that the solution, y, of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x}\sinh x + 4y\cosh x = 8\mathrm{e}^x \qquad (*)$$

satisfies $y = 3$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 4$ when $x = \ln 2$.

(a) **(i)** Find the Taylor series expansion for y about $x = \ln 2$ up to and including the quadratic term. [5]

(ii) Deduce an approximation for y when $x = 0.75$. Give your answer to 3 decimal places. [1]

Three students try different methods to calculate approximations for the value of y when $x = 0.75$. They do this by replacing $\sinh x$, $\cosh x$ and e^x in $(*)$ by the first few terms of their Maclaurin series and getting an approximate differential equation which they hope to be able to solve instead.

The first student uses quadratic approximations to $\sinh x$, $\cosh x$ and e^x; the second student uses linear approximations; and the third student uses constant approximations.

(b) **(i)** Find the approximate differential equations obtained by the three students. [4]

(ii) For the approximate differential equation obtained by the second student, find a particular integral. [3]

(iii) Solve the approximate differential equation obtained by the third student and use your answer to calculate a second approximation for the value of y when $x = 0.75$. Show full working and give the final answer correct to 3 decimal places. [9]