UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper for the guidance of teachers

9709 MATHEMATICS

9709/61 Paper 61, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

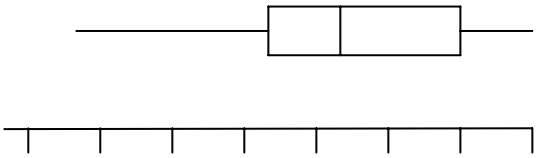
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$20p = 1.6 \quad p = 0.08$	M1 A1		Equation relating $20p$ to the mean Correct p can be implied
	$P(X > 2) = 1 - \{(0.92)^{20}$ $+ {}^{20}C_1(0.08)(0.92)^{19}$ $+ {}^{20}C_2\,(0.08)^2\,(0.92)^{18}\}$	M1		Bin expression involving $p^x(1-p)^{20-x}\,{}^{20}C_x$ any p
	$= 1 - (0.1887 + 0.3281 + 0.2711)$	M1		Subtracting 2 or 3 binomial probs from 1, one of which is P(0)
	$= 0.212$	A1	**[5]**	Correct answer
2	**(i)** $-0.16 - p + 0.16 + 2q + 0.66 = 1.05$	M1		Attempt at $\Sigma px = 1.05$ no dividing
	$-p + 2q = 0.39$	A1		Correct simplified equation
	$p + q = 0.42$	B1		Accept $p = 0.42 - q$ oe
	$q = 0.27$ $p = 0.15$	A1	**[4]**	Both answers correct
	(ii) $\text{Var}(X) = 4 \times 0.08 + p + 0.16 + 4q$ $+ 1.98 - (1.05)^2$	M1		Subst in $\Sigma px^2 - \text{mean}^2$ formula, mean^2 subt numerically, p +ve and < 1
	$= 2.59$	A1	**[2]**	Correct answer
3	**(i)** $P(85 < x < 100)$ $= 0.5 - P\left(z < \left(\frac{85-100}{7}\right)\right)$	B1		$\pm\frac{85-100}{7}$ seen oe or ± 2.14
	$= 0.5 - P\,(z < -2.143)$ $= 0.5 - (1 - \Phi(2.143))$ $= 0.9839 - 0.5$	M1		$\Phi - 0.5$
	$= 0.484$	A1	**[3]**	Correct answer rounding to
	(ii) $z = \Phi^{-1}(0.67) = 0.44$	B1		± 0.44 seen
	$0.44 = \frac{a-100}{7}$	M1		Standardising, with or without sq rt, no cc, no 7^2 must be z-value e.g. could be 0.412 or 0.413
	103.1 min (103) = upper limit	A1		Correct upper or lower boundary allow even if obtained from $z = 0.412$
	96.9 min = lower limit	A1	**[4]**	Correct other boundary

4	**(i)** 67	B1 **[1]**	
	(ii) LQ = 64 Med = 73 UQ = 90  	M1 B1 B1 B1 B1 **[5]**	Attempt to find all 3 quartiles can be implied Correct end whiskers (not dots or boxes), not through box, must look accurate Correct median line in box must look accurate Correct box ends must look accurate Correct uniform scale from at least 33 to 99, and label 'books' oe can be seen in title or scale
	(iii) books are fatter/ wider, or standard deviation /IQ range of the number of books per shelf is less	B1 **[1]**	Any sensible comment about width of books or s.d / IQ range not mean/median.
5	**(a)** **(i)** $1 \times 5 \times 4 \times 3$ or ${}^5C_3 \times 3!$ or ${}^5P_3$ = 60	M1 A1 **[2]**	One of these oe Correct final answer
	(ii) $1 \times 6^3 = 216$	M1 A1 **[2]**	Seeing 6^3 Correct answer
	(b) **(i)** 5G 0B = ${}^8C_5 = 56$ ($\times {}^6C_0$) 4G 1B = ${}^8C_4 \times {}^6C_1 = 420$ 3G 2B = ${}^8C_3 \times {}^6C_2 = 840$ total = 1316	M1 B1 A1 A1 **[4]**	Σ 2 or three 2-factor products, C or P Any correct option unsimplified A second correct option unsimplified Correct answer
	(ii) ${}^{11}C_2 + {}^{11}C_5$ = 55 + 462 = 517 OR cousins in P(3B, 2G) + P(4B, 1G) + P(5B, 0G) + cousins out P(3B, 2G) + P(2B, 3G) + P(1B, 4G) + P(0B, 5G) = 28 + 24 + 3 + 28 + 168 + 210 + 56 = 517	M1 B1 A1 M1 B1 A1 **[3]**	Adding two single perm or comb options ${}^{11}C_x + {}^{11}C_y$ One correct unsimplified option Correct answer Σ 5 or more 2-factor perm or comb terms 3 or more correct unsimplified options Correct answer

6 **(i)**	$\dfrac{{}^4C_2 \times {}^7C_1}{{}^{11}C_3} = 0.255$	M1	Using 2 combs mult for numerator and 1 comb for denom
		M1	Correct denom or num unsimplified
		A1	Correct answer
	OR $\dfrac{4}{11} \times \dfrac{3}{10} \times \dfrac{7}{9} \times 3$	M1	Multiplying 3 correct probs
		M1	Mult by 3 or Σ their 3 options
	= 0.255 (14/55) (42/165)	A1 **[3]**	Correct answer
(ii)	P(3rd is orange) = P(*P, P, O*) + P(*P, O, O*) + P(*O, P, O*) + P(*O, O, O*)	M1	Summing four 3-factor options with or without replacement
	$= \dfrac{4}{11} \times \dfrac{3}{10} \times \dfrac{7}{9} + \dfrac{4}{11} \times \dfrac{7}{10} \times \dfrac{6}{9} + \dfrac{7}{11} \times \dfrac{4}{10} \times \dfrac{6}{9} + \dfrac{7}{11} \times \dfrac{6}{10} \times \dfrac{5}{9}$	A1	At least 3 correct unsimplified options
	$= \left[\dfrac{14}{165} + \dfrac{28}{165} + \dfrac{28}{165} + \dfrac{7}{33}\right]$		
	= 7/11 (0.636	A1	Correct answer. Award B3 if the correct answer is stated with no working.
	OR using a tree diagram	**[3]**	
(iii)	$P(P \mid O) = \dfrac{P(P \cap O)}{P(O)}$	M1	Substituting in cond prob formula with at least one 3-factor product in num, and denom their **(ii)** or 7/11
	$= \dfrac{P(P,P,O) + P(P,O,O)}{P(O)}$	M1	Summing exactly 2 three-factor products in num
	$= \dfrac{28/110}{7/11} = \dfrac{28}{70} = \dfrac{4}{10} = 0.4$	A1 **[3]**	Correct answer
(iv)	$\mu = 121 \times \dfrac{4}{11} = 44$	B1	44 and 28 or 5.29 seen
	$\sigma^2 = 121 \times \dfrac{4}{11} \times \dfrac{7}{11} = 28$	M1	Standardising, with or without cc, must have sq rt on denom
	$P(X < 39) = \Phi\left(\dfrac{38.5 - 44}{\sqrt{28}}\right)$	M1	cc either 39.5 or 38.5
	$= \Phi(-1.039)$	M1	Correct area "1 – Φ" seen
	= 1 – 0.8506		
	= 0.149	A1 **[5]**	Correct answer