Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/51**

Paper 5 Mechanics 2 **(M2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **4** printed pages and **1** insert.

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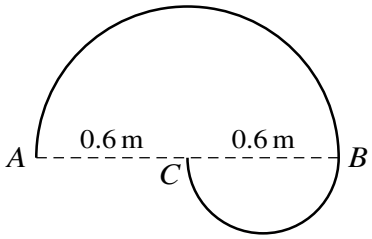
[Turn over

1 A particle P of mass 0.3 kg moves in a circle with centre O on a smooth horizontal surface. P is attached to O by a light elastic string of modulus of elasticity 12 N and natural length l m. The speed of P is 4 m s^{-1}, and the radius of the circle in which it moves is $2l$ m. Calculate l. [4]

2



A uniform wire is bent to form an object which has a semicircular arc with diameter AB of length 1.2 m, with a smaller semicircular arc with diameter BC of length 0.6 m. The end C of the smaller arc is at the centre of the larger arc (see diagram). The two semicircular arcs of the wire are in the same plane.

(i) Show that the distance of the centre of mass of the object from the line ACB is 0.191 m, correct to 3 significant figures. [3]

The object is freely suspended at A and hangs in equilibrium.

(ii) Find the angle between ACB and the vertical. [4]

3 A small block B of mass 0.25 kg is released from rest at a point O on a smooth horizontal surface. After its release the velocity of B is v m s^{-1} when its displacement is x m from O. The force acting on B has magnitude $(2 + 0.3x^2)$ N and is directed horizontally away from O.

(i) Show that $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 1.2x^2 + 8$. [2]

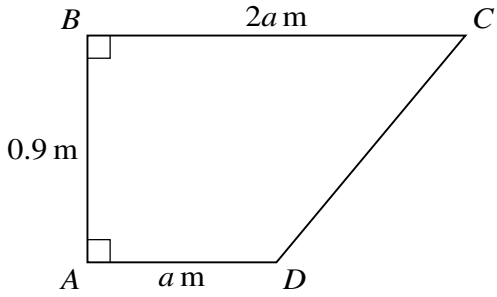
(ii) Find the velocity of B when $x = 1.5$. [3]

An extra force acts on B after $x = 1.5$. It is given that, when $x > 1.5$,

$$v\frac{\mathrm{d}v}{\mathrm{d}x} = 1.2x^2 + 6 - 3x.$$

(iii) Find the magnitude of this extra force and state the direction in which it acts. [2]

4



The diagram shows the cross-section $ABCD$ through the centre of mass of a uniform solid prism. $AB = 0.9$ m, $BC = 2a$ m, $AD = a$ m and angle ABC = angle $BAD = 90°$.

(i) Calculate the distance of the centre of mass of the prism from AD. [2]

(ii) Express the distance of the centre of mass of the prism from AB in terms of a. [2]

The prism has weight 18 N and rests in equilibrium on a rough horizontal surface, with AD in contact with the surface. A horizontal force of magnitude 6 N is applied to the prism. This force acts through the centre of mass in the direction BC.

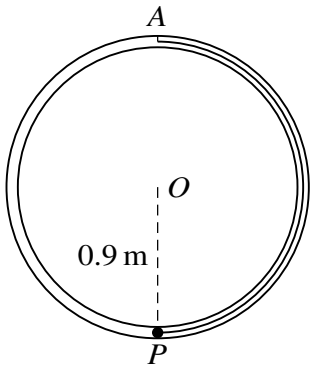
(iii) Given that the prism is on the point of toppling, calculate a. [3]

5 A small ball B of mass 0.4 kg moves in a horizontal circle with centre O and radius 0.6 m on a smooth horizontal surface. One end of a light inextensible string is attached to B; the other end of the string is attached to a fixed point 0.45 m vertically above O.

(i) Given that the tension in the string is 5 N, calculate the speed of B. [3]

(ii) Find the greatest possible tension in the string for the motion, and the corresponding angular speed of B. [4]

6



The diagram shows a smooth narrow tube formed into a fixed vertical circle with centre O and radius 0.9 m. A light elastic string with modulus of elasticity 8 N and natural length 1.2 m has one end attached to the highest point A on the inside of the tube. The other end of the string is attached to a particle P of mass 0.2 kg. The particle is released from rest at the lowest point on the inside of the tube. By considering energy, calculate

(i) the speed of P when it is at the same horizontal level as O, [4]

(ii) the speed of P at the instant when the string becomes slack. [3]

[Turn over

7 A particle P is projected with speed $35\ \text{m s}^{-1}$ from a point O on a horizontal plane. In the subsequent motion, the horizontal and vertically upwards displacements of P from O are x m and y m respectively. The equation of the trajectory of P is

$$y = kx - \frac{(1 + k^2)x^2}{245},$$

where k is a constant. P passes through the points $A\,(14,\ a)$ and $B\,(42,\ 2a)$, where a is a constant.

(i) Calculate the two possible values of k and hence show that the larger of the two possible angles of projection is 63.435°, correct to 3 decimal places. [5]

For the larger angle of projection, calculate

(ii) the time after projection when P passes through A, [2]

(iii) the speed and direction of motion of P when it passes through B. [4]