



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Advanced Level

**MATHEMATICS**

**9709/51**

Paper 5 Mechanics 2 (M2)

**October/November 2013**

**1 hour 15 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

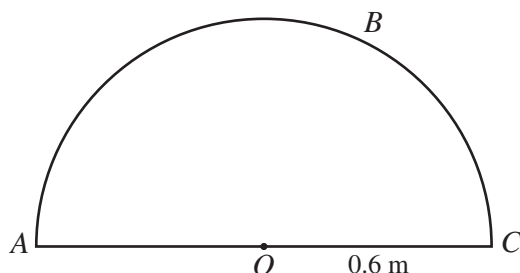
Answer **all** the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
Where a numerical value for the acceleration due to gravity is needed, use  $10 \text{ m s}^{-2}$ .  
The use of an electronic calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 50.  
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

- 1 A particle  $P$  of mass  $0.3 \text{ kg}$  is attached to one end of a light inextensible string of length  $0.6 \text{ m}$ . The other end of the string is attached to a fixed point  $O$  of a smooth horizontal plane.  $P$  moves on the plane at constant speed  $5 \text{ m s}^{-1}$  in a circle with centre  $O$ . Calculate the tension in the string. [2]

2



A uniform frame consists of a semicircular arc  $ABC$  of radius  $0.6 \text{ m}$  together with its diameter  $AOC$ , where  $O$  is the centre of the semicircle (see diagram).

- (i) Calculate the distance of the centre of mass of the frame from  $O$ . [4]

The frame is freely suspended at  $A$  and hangs in equilibrium.

- (ii) Calculate the angle between  $AC$  and the vertical. [2]

- 3 A particle  $P$  of mass  $0.8 \text{ kg}$  moves along the  $x$ -axis on a horizontal surface. When the displacement of  $P$  from the origin  $O$  is  $x \text{ m}$  the velocity of  $P$  is  $v \text{ m s}^{-1}$  in the positive  $x$ -direction. Two horizontal forces act on  $P$ . One force has magnitude  $4e^{-x} \text{ N}$  and acts in the positive  $x$ -direction. The other force has magnitude  $2.4x^2 \text{ N}$  and acts in the negative  $x$ -direction.

- (i) Show that  $v \frac{dv}{dx} = 5e^{-x} - 3x^2$ . [2]

- (ii) The velocity of  $P$  as it passes through  $O$  is  $6 \text{ m s}^{-1}$ . Find the velocity of  $P$  when  $x = 2$ . [5]

- 4 A small ball  $B$  is projected from a point  $O$  with speed  $14 \text{ m s}^{-1}$  at an angle of  $60^\circ$  above the horizontal.

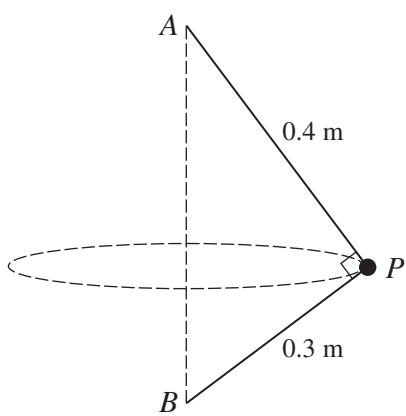
- (i) Calculate the speed and direction of motion of  $B$  for the instant  $1.8 \text{ s}$  after projection. [5]

The point  $O$  is  $2 \text{ m}$  above a horizontal plane.

- (ii) Calculate the time after projection when  $B$  reaches the plane. [3]

3

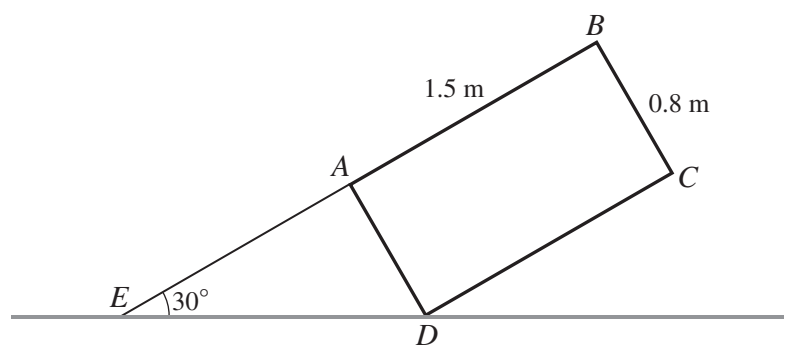
5



A particle  $P$  of mass  $0.2$  kg is attached to a fixed point  $A$  by a light inextensible string of length  $0.4$  m. A second light inextensible string of length  $0.3$  m connects  $P$  to a fixed point  $B$  which is vertically below  $A$ . The particle  $P$  moves in a horizontal circle, which has its centre on the line  $AB$ , with the angle  $APB = 90^\circ$  (see diagram).

- (i) Given that the tensions in the two strings are equal, calculate the speed of  $P$ . [5]
- (ii) It is given instead that  $P$  moves with its least possible angular speed for motion in this circle. Find this angular speed. [3]

6



$ABCD$  is the cross-section through the centre of mass of a uniform rectangular block of weight  $260$  N. The lengths  $AB$  and  $BC$  are  $1.5$  m and  $0.8$  m respectively. The block rests in equilibrium with the point  $D$  on a rough horizontal floor. Equilibrium is maintained by a light rope attached to the point  $A$  on the block and the point  $E$  on the floor. The points  $E$ ,  $A$  and  $B$  lie in a straight line inclined at  $30^\circ$  to the horizontal (see diagram).

- (i) By taking moments about  $D$ , show that the tension in the rope is  $146$  N, correct to 3 significant figures. [5]
- (ii) Given that the block is in limiting equilibrium, calculate the coefficient of friction between the block and the floor. [4]

[Question 7 is printed on the next page.]

7 A particle  $P$  of mass  $0.4$  kg is attached to one end of a light elastic string of natural length  $0.8$  m and modulus of elasticity  $32$  N. The other end of the string is attached to a fixed point  $O$ . The particle is released from rest at  $O$ .

(i) Calculate the distance  $OP$  at the instant when  $P$  first comes to instantaneous rest. [4]

A horizontal plane is fixed at a distance  $1$  m below  $O$ . The particle  $P$  is again released from rest at  $O$ .

(ii) Calculate the speed of  $P$  immediately before it collides with the plane. [3]

(iii) In the collision with the plane,  $P$  loses  $96\%$  of its kinetic energy. Calculate the distance  $OP$  at the instant when  $P$  first comes to instantaneous rest above the plane, given that this occurs when the string is slack. [3]