

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level
Advanced International Certificate of Education

MARK SCHEME for the November 2003 question papers

MATHEMATICS

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| 9709/01 | Paper 1 (Pure 1), maximum raw mark 75 |
| 9709/02 | Paper 2 (Pure 2), maximum raw mark 50 |
| 9709/03, 8719/03 | Paper 3 (Pure 3), maximum raw mark 75 |
| 9709/04 | Paper 4 (Mechanics 1), maximum raw mark 50 |
| 9709/05, 8719/05 | Paper 5 (Mechanics 2), maximum raw mark 50 |
| 9709/06, 0390/06 | Paper 6 (Probability and Statistics 1), maximum raw mark 50 |
| 9709/07, 8719/07 | Paper 7 (Probability and Statistics 2), maximum raw mark 50 |

These mark schemes are published as an aid to teachers and students, to indicate the requirements of the examination. They show the basis on which Examiners were initially instructed to award marks. They do not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2003 question papers for most IGCSE and GCE Advanced Level syllabuses.

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Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
 - When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol \surd implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.
- The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.
- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
 - For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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- The following abbreviations may be used in a mark scheme or used on the scripts:

| | |
|-----|---|
| AEF | Any Equivalent Form (of answer is equally acceptable) |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

Penalties

- MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

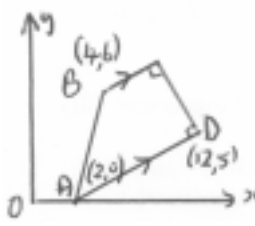
CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

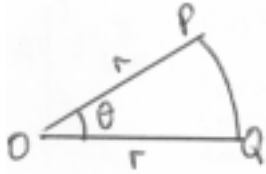
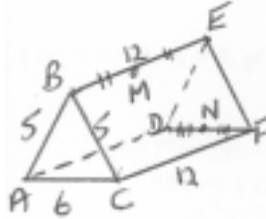
GCE A AND AS LEVEL

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| MARK SCHEME |
| MAXIMUM MARK: 75 |
| SYLLABUS/COMPONENT: 9709/01 MATHEMATICS Pure Mathematics : Paper One |

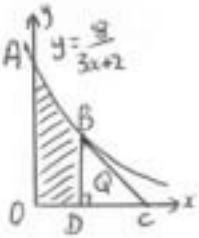
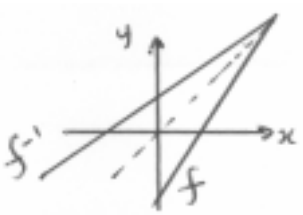
| Page 1 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 1 |

| | | |
|--|---|---|
| <p>1</p> $x(11-2x) = 12$ $2x^2 - 11x + 12 = 0$ <p>Solution of quadratic → (1½, 8) and (4, 3)</p> | <p>M1 A1 DM1 A1 [4]</p> | <p>Complete elimination of x, or of y. Correct quadratic. (or $y^2 - 11y + 24 = 0$) Correct method of solution → 2 values All correct (guesswork or TI B1 for one pair of values, full marks for both)</p> |
| <p>2</p> <p>(i) $4s^4 + 5 = 7(1 - s^2) \rightarrow 4x^2 + 7x - 2 = 0$</p> <p>(ii) $4s^4 + 7s^2 - 2 = 0$ → $s^2 = \frac{1}{4}$ or $s^2 = -2$ → $\sin\theta = \pm\frac{1}{2}$ → $\theta = 30^\circ$ and 150° and $\theta = 210^\circ$ and 330°</p> | <p>B1 [1] M1 A1A1√ A1√ [4]</p> | <p>Use of $s^2 + c^2 = 1$. Answer given. Recognition of quadratic in s^2 Co. For 180° - "his value" For other 2 answers from "his value", providing no extra answers in the range or answers from $s^2 = -1$</p> |
| <p>3</p> <p>(a) $a = 60, n = 48, S_n = 3726$ S_n formula used → $d = \\$0.75$ 3^{rd} term = $a + 2d = \\$61.50$</p> <p>(b) $a = 6, ar = 4 \therefore r = \frac{2}{3}$ $S_\infty = a/(1-r) = 18$</p> | <p>M1 A1 A1√ [3] M1 M1A1 [3]</p> | <p>Correct formula (M0 if nth term used) Co Use of $a + 2d$ with his d. 61.5 ok. a, ar correct, and r evaluated Correct formula used, but needs $r < 1$ for M mark</p> |
| <p>4</p> <p>(i) $y = x^3 - 2x^2 + x + c$ (1, 5) used to give $c = 5$</p> <p>(ii) $3x^2 - 4x + 1 > 0$ → end values of 1 and $\frac{1}{3}$ → $x < \frac{1}{3}$ and $x > 1$</p> | <p>B2,1,0 B1√ [3] M1 A1 A1 [3]</p> | <p>Co - unsimplified ok. Must have integrated + use of $x = 1$ and $y = 5$ for c Set to 0 and attempt to solve. Co for end values – even if $<, >, =$, etc Co (allow \leq and \geq). Allow $1 < x < \frac{1}{3}$</p> |
| <p>5</p> <p>(i) m of BC = $\frac{1}{2}$ Eqn BC $y - 6 = \frac{1}{2}(x - 4)$ m of CD = -2 eqn CD $y - 5 = -2(x - 12)$</p>  <p>(ii) Sim eqns $2y = x + 8$ and $y + 2x = 29$ → C (10, 9)</p> | <p>B1 M1A1√ M1 A1√ [5] M1 A1 [2]</p> | <p>Co Correct form of eqn. √ on $m = \frac{1}{2}$. Use of $m_1 m_2 = -1$ √ on his "$\frac{1}{2}$" but needs both M marks. Method for solving Co Diagram only for (ii), allow B1 for (10, 9)</p> |

| Page 2 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 1 |

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|--|---|---|
| <p>6</p>  <p>(i) $20 = 2r + r\theta$ $\rightarrow \theta = (20 / r) - 2$</p> <p>(ii) $A = \frac{1}{2}r^2\theta$ $\rightarrow A = 10r - r^2$</p> <p>(iii) Cos rule $PQ^2 = 8^2 + 8^2 - 2 \cdot 8 \cdot 8 \cos 0.5$</p> <p>Or trig $PQ = 2 \times 8 \sin 0.25$ $\rightarrow PQ = 3.96$ (allow 3.95).</p> | <p>M1 A1 [2]</p> <p>M1 A1 [2]</p> <p>M1 A1 [3]</p> | <p>Eqn formed + use of $r\theta$ + at least one r Answer given.</p> <p>Appropriate use of $\frac{1}{2}r^2\theta$ Co – but ok unsimplified – eg $\frac{1}{2}r^2(20/r) - 2$</p> <p>Recognition of “chord” + any attempt at trigonometry in triangle. Correct expression for PQ or PQ^2.</p> <p>Co</p> |
| <p>7</p>  <p>(i) Height = 4</p> <p>(ii) $\mathbf{MC} = 3\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$ $\mathbf{MN} = 6\mathbf{j} - 4\mathbf{k}$</p> <p>(iii) $\mathbf{MC} \cdot \mathbf{MN} = -36 + 16 = -20$ $\mathbf{MC} \cdot \mathbf{MN} = \sqrt{61} \sqrt{52} \cos \theta$ $\rightarrow \theta = 111^\circ$</p> | <p>B1 [1]</p> <p>B2,1√ B1√ [3]</p> <p>M1A1√ M1 A1 [4]</p> | <p>Pythagoras or guess – anywhere, 4k ok.</p> <p>√ for “4”. Special case B1 for $-3\mathbf{i} + 6\mathbf{j} + 4\mathbf{k}$ √ on “4”. Accept column vectors.</p> <p>(nb if (ii) incorrect, but answers are correct in (iii) allow feedback).</p> <p>Use of $x_1y_1 + x_2y_2 + x_3y_3$. √ on \mathbf{MC} and \mathbf{MN} Product of two moduli and $\cos \theta$. Co.</p> <p>Nb If both \mathbf{MC} and \mathbf{MN} “reversed”, allow 111° for full marks.</p> |
| <p>8</p> <p>(i) $y = 72 \div (2x^2)$ or $36 \div x^2$ $A = 4x^2 + 6xy$ $\rightarrow A = 4x^2 + 216 \div x$</p> <p>(ii) $dA/dx = 8x - 216 \div x^2$ $= 0$ when $8x^3 = 216$ $\rightarrow x = 3$</p> <p>(iii) Stationary value = 108 cm^2</p> <p>$d^2A/dx^2 = 8 + 432 \div x^3$ \rightarrow Positive when $x = 3$ Minimum.</p> | <p>B1 M1 A1 [3]</p> <p>M1 DM1 A1 [3]</p> <p>A1√ M1 A1 [3]</p> | <p>Co from volume = l b h . Attempts most of the faces (4 or more) Co – answer was given.</p> <p>Reasonable attempt at differentiation. Sets his differential to 0 and uses. Co. (answer = ± 3 loses last A mark)</p> <p>For putting his x into his A. Allow in (ii).</p> <p>Correct method – could be signs of dA/dx A mark needs d^2A/dx^2 correct algebraically, + $x = 3$ + minimum. It does not need “24”.</p> |

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| <p>9</p>  <p>(i) $dy/dx = -24/(3x+2)^2$</p> <p>Eqn of tangent $y-1 = -3/8(x-2)$ Cuts $y=0$ when $x=4\frac{2}{3}$</p> <p>Area of Q = $\frac{1}{2} \times 2\frac{2}{3} \times 1 = \frac{4}{3}$</p> <p>(ii) $Vol = \pi \int y^2 dx = \pi \int 64(3x+2)^{-2} dx$ $= \pi [-64(3x+2)^{-1} \div 3]$ Limits from 0 to 2 $\rightarrow 8\pi$</p> | <p>M1A1</p> <p>M1A1√</p> <p>M1A1 [6]</p> <p>M1 A1A1 DM1 A1 [5]</p> | <p>Use of fn of fn. Needs $\times 3$ for M mark. Co.</p> <p>Use of line form with dy/dx. Must use calculus. \sqrt on his dy/dx. Normal M0.</p> <p>Needs $y=0$ and $\frac{1}{2}bh$ for M mark. (beware fortuitous answers)</p> <p>Uses $\int y^2 +$ some integration $\rightarrow (3x+2)^k$. A1 without the $\div 3$. A1 for $\div 3$ and π Correct use of 0 and 2. DMO if 0 ignored. Co. Beware fortuitous answers.</p> |
| <p>10</p> <p>(i) $fg(x) = g$ first, then f $= 8/(2-x) - 5 = 7$ $\rightarrow x = 1\frac{1}{3}$</p> <p>(or $f(A)=7, A=6, g(x)=6, \rightarrow x = 1\frac{1}{3}$)</p> <p>(ii) $f^{-1} = \frac{1}{2}(x+5)$ Makes y the subject $y = 4 \div (2-x)$ $\rightarrow g^{-1} = 2 - (4 \div x)$</p> <p>(iii) $2 - 4/x = \frac{1}{2}(x+5)$ $\rightarrow x^2 + x + 8 = 0$ Use of $b^2 - 4ac \rightarrow$ Negative value \rightarrow No roots.</p> <p>(iv)</p>  | <p>M1 DM1 A1 [3]</p> <p>B1 M1 A1 [3]</p> <p>M1 M1 A1 [3]</p> <p>B1 B1 B1 [3]</p> | <p>Correct order - g first, then into f. Correct method of solution of $fg=7$. Co. (nb gf gets 0/3) (M1 for 6. M1 for $g(x)=6$. A1)</p> <p>Anywhere in the question. For changing the subject. Co - any correct answer. (A0 if $f(y)$.)</p> <p>Algebra leading to a quadratic. Quadratic=0 + use of $b^2 - 4ac$. Correct deduction from correct quadratic.</p> <p>Sketch of f Sketch of f^{-1} Evidence of symmetry about $y=x$.</p> |

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/02

MATHEMATICS
Pure Mathematics : Paper Two



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| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 2 |

- 1** *EITHER:* State or imply non-modular inequality e.g. $-2 < 8-3x < 2$, or $(8-3x)^2 < 2^2$,
or corresponding equation or pair of equations M1
Obtain critical values 2 and $3\frac{1}{3}$ A1
State correct answer $2 < x < 3\frac{1}{3}$ A1
- OR:* State one critical value (probably $x = 2$), from a graphical method or by
inspection or by solving a linear equality or equation B1
State the other critical value correctly B1
State correct answer $2 < x < 3\frac{1}{3}$ B1
- [3]
- 2** State or imply at any stage $\ln y = \ln k - x \ln a$ B1
Equate estimate of $\ln y$ - intercept to $\ln k$ M1
Obtain value for k in the range 9.97 ± 0.51 A1
Calculate gradient of the line of data points M1
Obtain value for a in the range 2.12 ± 0.11 A1
- [5]
- 3 (i) EITHER:** Substitute -1 for x and equate to zero M1
Obtain answer $a=6$ A1
- OR:* Carry out complete division and equate remainder to zero M1
Obtain answer $a=6$ A1
- [2]
- (ii)** Substitute 6 for a and either show $f(x) = 0$ or divide by $(x - 2)$ obtaining a
remainder of zero B1
- EITHER:* State or imply $(x + 1)(x - 2) = x^2 - x - 2$ B1
Attempt to find another quadratic factor by division or inspection M1
State factor $(x^2 + x - 3)$ A1
- OR:* Obtain $x^3 + 2x^2 - 2x - 3$ after division by $x + 1$, or $x^3 - x^2 - 5x + 6$
after division by $x - 2$ B1
Attempt to find a quadratic factor by further division by relevant divisor
or by inspection M1
State factor $(x^2 + x - 3)$ A1
- [4]
- 4 (i)** State answer $R = 2$ B1
Use trig formula to find α M1
Obtain answer $\alpha = \frac{1}{3}\pi$ A1
- [3]

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| (ii) | Carry out, or indicate need for, evaluation of $\cos^{-1}(\sqrt{2}/2)$ | M1* |
| | Obtain, or verify, the solution $\theta = \frac{7}{12}\pi$ | A1 |
| | Attempt correct method for the other solution in range i.e. $-\cos^{-1}(\sqrt{2}/2) + \alpha$ | M1(dep*) |
| | Obtain solution $\theta = \frac{1}{12}\pi$: [M1A0 for $\frac{25\pi}{12}$] | A1 |
| | | [4] |
| 5 (i) | Make recognisable sketch of $y = 2^x$ or $y = x^2$, for $x < 0$ | B1 |
| | Sketch the other graph correctly | B1 |
| | | [2] |
| (ii) | Consider sign of $2^x - x^2$ at $x = -1$ and $x = -0.5$, or equivalent | M1 |
| | Complete the argument correctly with appropriate calculations | A1 |
| | | [2] |
| (iii) | Use the iterative form correctly | M1 |
| | Obtain final answer -0.77 | A1 |
| | Show sufficient iterations to justify its accuracy to 2 s.f., or show there is a sign change in the interval $(-0.775, -0.765)$ | A1 |
| | | |
| | | [3] |
| 6 (i) | State A is $(4, 0)$ | B1 |
| | State B is $(0, 4)$ | B1 |
| | | [2] |
| (ii) | Use the product rule to obtain the first derivative | M1(dep) |
| | Obtain derivative $(4 - x)e^x - e^x$, or equivalent | A1 |
| | Equate derivative to zero and solve for x | M1 (dep) |
| | Obtain answer $x = 3$ only | A1 |
| | | [4] |
| (iii) | Attempt to form an equation in p e.g. by equating gradients of OP and the tangent at P , or by substituting $(0, 0)$ in the equation of the tangent at P | M1 |
| | Obtain equation in any correct form e.g. $\frac{4-p}{p} = 3 - p$ | A1 |
| | Obtain 3-term quadratic $p^2 - 4p + 4 = 0$, or equivalent | A1 |
| | Attempt to solve a quadratic equation in p | M1 |
| | Obtain answer $p = 2$ only | A1 |
| | | |
| | | [5] |
| 7 (i) | Attempt to differentiate using the quotient, product or chain rule | M1 |
| | Obtain derivative in any correct form | A1 |
| | Obtain the given answer correctly | A1 |
| | | |
| | | [3] |

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| (ii) | State or imply the indefinite integral is $-\cot x$ | B1 |
| | Substitute limits and obtain given answer correctly | B1 |
| | | [2] |
| (iii) | Use $\cot^2 x = \operatorname{cosec}^2 x - 1$ and attempt to integrate both terms, or equivalent | M1 |
| | Substitute limits where necessary and obtain a correct unsimplified answer | A1 |
| | Obtain final answer $\sqrt{3} - \frac{1}{3}\pi$ | A1 |
| | | [3] |
| (iv) | Use $\cos 2A$ formula and reduce denominator to $2\sin^2 x$ | B1 |
| | Use given result and obtain answer of the form $k\sqrt{3}$ | M1 |
| | Obtain correct answer $\frac{1}{2}\sqrt{3}$ | A1 |
| | | [3] |

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GCE A AND AS LEVEL

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| MARK SCHEME |
| MAXIMUM MARK: 75 |
| SYLLABUS/COMPONENT: 9709/03, 8719/03 MATHEMATICS Mathematics and Higher Mathematics : Paper 3 |

| Page 1 | Mark Scheme | Syllabus | Paper |
|--------|--------------------------------|-----------|-------|
| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 3 |

- 1 *EITHER:* State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation B1
 Use correct method for solving an equation of the form $2^x = a$ M1
 Obtain critical values 1.58 and 3.70, or exact equivalents A1
 State correct answer $1.58 < x < 3.70$ A1
- OR:* Use correct method for solving an equation of the form $2^x = a$ M1
 Obtain one critical value (probably 3.70), or exact equivalent A1
 Obtain the other critical value, or exact equivalent A1
 State correct answer $1.58 < x < 3.70$ A1
- [4]**

[Allow 1.59 and 3.7. Condone \leq for $<$. Allow final answers given separately. Exact equivalents must be in terms of ln or logarithms to base 10.]

[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

- 2 *EITHER:* Obtain correct unsimplified version of the x^2 or x^4 term of the expansion of $(1 + \frac{1}{2}x^2)^{-2}$ or $(2 + x^2)^{-2}$ M1
 State correct first term $\frac{1}{4}$ B1
 Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\binom{-2}{1}$.]

[SR: Answers given as $\frac{1}{4}(1 - x^2 + \frac{3}{4}x^4)$ earn M1B1A1.]

[SR: Solutions involving $k(1 + \frac{1}{2}x^2)^{-2}$, where $k = 2, 4$ or $\frac{1}{2}$ can earn M1 and A1 for a correct simplified term in x^2 or x^4 .]

- OR:* Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = kx(2 + x^2)^{-3}$ M1
 State correct first term $\frac{1}{4}$ B1
 Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

- 3 Use correct $\cos 2A$ formula, or equivalent pair of correct formulas, to obtain an equation in $\cos \theta$ M1
 Obtain 3-term quadratic $6 \cos^2 \theta + \cos \theta - 5 = 0$, or equivalent A1
 Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$ M1
 Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians A1
 Obtain answer 180° or π (or 3.14) radians and no others in range A1

[The answer $\theta = 180^\circ$ found by inspection can earn B1.]

[Ignore answers outside the given range.]

[5]

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| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 3 |

| | | |
|---|---|-------|
| 4(i) EITHER | Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}} \frac{dy}{dx}$, or equivalent | B1+B1 |
| | Obtain answer in any correct form, e.g. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$ | B1 |
| OR: | Using chain or product rule, differentiate $(\sqrt{a} - \sqrt{x})^2$ | M1 |
| | Obtain derivative in any correct form | A1 |
| | Express $\frac{dy}{dx}$ in terms of x and y only in any correct form | A1 |
| OR: | Expand $(\sqrt{a} - \sqrt{x})^2$, differentiate and obtain term $-2 \cdot \frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent | B1 |
| | Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$ | B1 |
| | Express $\frac{dy}{dx}$ in terms of x and y only in any correct form | B1 |
| | | [3] |
| (ii) | State or imply coordinates of P are $(\frac{1}{4}a, \frac{1}{4}a)$ | B1 |
| | Form equation of the tangent at P | M1 |
| | Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent | A1 |
| | | [3] |
| 5 (i) | Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{1}{2}\pi$ | B1 |
| | Sketch the other graph correctly and justify the given statement | B1 |
| | | [2] |
| [Award B1 for a sketch with positive y -intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3 - x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.] | | |
| (ii) | State or imply equation $\alpha = \cos^{-1}(1/(3 - \alpha^2))$ or $\cos \alpha = 1/(3 - \alpha^2)$ | B1 |
| | Rearrange this in the form given in part (i) i.e. $\sec \alpha = 3 - \alpha^2$ | B1 |
| | | [2] |
| [Or work <i>vice versa</i> .] | | |
| (iii) | Use the iterative formula with $0 \leq x_1 \leq \sqrt{2}$ | M1 |
| | Obtain final answer 1.03 | A1 |
| | Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035) | A1 |
| | | [3] |

| Page 3 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 3 |

- 6 (i) Use product or quotient rule to find derivative M1
 Obtain derivative in any correct form A1
 Equate derivative to zero and solve a linear equation in x M1
 Obtain answer $3\frac{1}{2}$ only A1
[4]
- (ii) State first step of the form $\pm\frac{1}{2}(3-x)e^{-2x} \pm\frac{1}{2}\int e^{-2x}dx$, with or without 3 M1
 State correct first step e.g. $-\frac{1}{2}(3-x)e^{-2x} -\frac{1}{2}\int e^{-2x}dx$, or equivalent, with or without 3 A1
 Complete the integration correctly obtaining $-\frac{1}{2}(3-x)e^{-2x} +\frac{1}{4}e^{-2x}$, or equivalent A1
 Substitute limits $x=0$ and $x=3$ correctly in the complete integral M1
 Obtain answer $\frac{1}{4}(5+e^{-6})$, or exact equivalent (allow e^0 in place of 1) A1
[5]
- 7 (i) *EITHER*: Attempt multiplication of numerator and denominator by $3+2i$, or equivalent M1
 Simplify denominator to 13 or numerator to $13+26i$ A1
 Obtain answer $u=1+2i$ A1
- OR*: Using correct processes, find the modulus and argument of u M1
 Obtain modulus $\sqrt{5}$ (or 2.24) or argument $\tan^{-1}2$ (or 63.4° or 1.11 radians) A1
 Obtain answer $u=1+2i$ A1
[3]
- (ii) Show the point U on an Argand diagram in a relatively correct position B1√
 Show a circle with centre U B1√
 Show a circle with radius consistent with 2 B1√
[3]
- [f.t. on the value of u .]
- (iii) State or imply relevance of the appropriate tangent from O to the circle B1√
 Carry out a complete strategy for finding max arg z M1
 Obtain final answer 126.9° (2.21 radians) A1
[3]
- [Drawing the appropriate tangent is sufficient for B1√.]
 [A final answer obtained by measurement earns M1 only.]

| Page 4 | Mark Scheme | Syllabus | Paper |
|--------|--------------------------------|-----------|-------|
| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 3 |

- 8 (i) EITHER:** Divide by denominator and obtain a quadratic remainder M1
 Obtain $A = 1$ A1
 Use any relevant method to obtain B, C or D M1
 Obtain one correct answer A1
 Obtain $B = -1, C = 2, D = 0$ A1

- OR:** Reduce *RHS* to a single fraction and identify numerator with that of $f(x)$ M1
 Obtain $A = 1$ A1
 Use any relevant method to obtain B, C or D M1
 Obtain one correct answer A1
 Obtain $B = -1, C = 2, D = 0$ A1

[5]

- (ii)** Integrate and obtain terms $x - \ln(x - 1)$, or equivalent B1√
 Obtain third term $\ln(x^2 + 1)$, or equivalent B1√
 Substitute correct limits correctly in the complete integral M1
 Obtain given answer following full and exact working A1

[4]

[If $B = 0$ the first B1√ is not available.]

[SR: If A is omitted in part (i), treat as if $A = 0$. Thus only M1M1 and B1√B1√M1 are available.]

- 9 (i)** Separate variables and attempt to integrate $\frac{1}{\sqrt{(P - A)}}$ M1
 Obtain term $2\sqrt{(P - A)}$ A1
 Obtain term $-kt$ A1

[3]

- (ii)** Use limits $P = 5A, t = 0$ and attempt to find constant c M1
 Obtain $c = 4\sqrt{A}$, or equivalent A1
 Use limits $P = 2A, t = 2$ and attempt to find k M1
 Obtain given answer $k = \sqrt{A}$ correctly A1

[4]

- (iii)** Substitute $P = A$ and attempt to calculate t M1
 Obtain answer $t = 4$ A1

[2]

- (iv)** Using answers to part (ii), attempt to rearrange solution to give P in terms of A and t M1
 Obtain $P = \frac{1}{4}A(4 + (4 - t)^2)$, or equivalent, having squared \sqrt{A} A1

[2]

[For the M1, $\sqrt{(P - A)}$ must be treated correctly.]

| Page 5 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 3 |

| | | |
|---------------------|--|------------|
| 10 (i) | Express general point of l or m in component form e.g. $(1 + 2s, s, -2 + 3s)$ or $(6 + t, -5 - 2t, 4 + t)$ | B1 |
| | Equate at least two corresponding pairs of components and attempt to solve for s or t | M1 |
| | Obtain $s = 1$ or $t = -3$ | A1 |
| | Verify that all three component equations are satisfied | A1 |
| | Obtain position vector $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ of intersection point, or equivalent | A1 |
| | | [5] |
| (ii) EITHER: | Use scalar product to obtain $2a + b + 3c = 0$ and $a - 2b + c = 0$ | B1 |
| | Solve and find one ratio e.g. $a : b$ | M1 |
| | State one correct ratio | A1 |
| | Obtain answer $a : b : c = 7 : 1 : -5$, or equivalent | A1 |
| | Substitute coordinates of a relevant point and values of a, b and c in general equation of plane and calculate d | M1 |
| | Obtain answer $7x + y - 5z = 17$, or equivalent | A1 |
| OR: | Using two points on l and one on m (or <i>vice versa</i>) state three simultaneous equations in a, b, c and d e.g. $3a + b + c = d, a - 2c = d$ and $6a - 5b + 4c = d$ | B1√ |
| | Solve and find one ratio e.g. $a : b$ | M1 |
| | State one correct ratio | A1 |
| | Obtain a ratio of three unknowns e.g. $a : b : c = 7 : 1 : -5$, or equivalent | A1 |
| | Use coordinates of a relevant point and found ratio to find fourth unknown e.g. d | M1 |
| | Obtain answer $7x + y - 5z = 17$, or equivalent | A1 |
| OR: | Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ | B1√ |
| | State 3 equations in x, y, z, λ and μ | M1 |
| | State 3 correct equations | A1√ |
| | Eliminate λ and μ | M1 |
| | Obtain equation in any correct unsimplified form | A1 |
| | Obtain $7x + y - 5z = 17$, or equivalent | A1 |
| OR: | Attempt to calculate vector product of vectors parallel to l and m | M1 |
| | Obtain two correct components of the product | A1 |
| | Obtain correct product, e.g. $7\mathbf{i} + \mathbf{j} - 5\mathbf{z}$ | A1 |
| | State that the plane has equation of the form $7x + y - 5z = d$ | A1√ |
| | Substitute coordinates of a relevant point and calculate d | M1 |
| | Obtain answer $7x + y - 5z = 17$, or equivalent | A1 |
| | [6] | |

[The follow through is on $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ only.]

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GCE A AND AS LEVEL

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| MARK SCHEME |
| MAXIMUM MARK: 50 |
| SYLLABUS/COMPONENT: 9709/04 MATHEMATICS Paper 4 (Mechanics 1) |

| Page 1 | Mark Scheme | Syllabus | Paper |
|--------|--------------------------------|----------|-------|
| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 4 |

| | | | | |
|---|-------|--|------|---|
| 1 | (i) | The force is 320 N | B1 | 1 |
| | (ii) | For using Newton's second law (3 terms needed) | M1 | |
| | | 320 – R = 100 × 0.5 | A1 √ | |
| | | Resistance is 270 N | A1 | 3 |
| 2 | (i) | Speed is 20 ms ⁻¹ | B1 | 1 |
| | (ii) | For using $s = \frac{1}{2}gt^2$ $45 = \frac{1}{2}10t^2$ | M1 | |
| | | Time taken is 3 s | A1 | 2 |
| | (iii) | For using $v^2 = u^2 + 2gs$ ($40^2 = 30^2 + 2 \times 10s$) | M1 | |
| | | Distance fallen is 35 m | A1 | 2 |
| 3 | (i) | For using the idea of work as a force times a distance ($25 \times 2 \cos 15^\circ$) | M1 | |
| | | Work done is 48.3 J | A1 | 2 |
| | (ii) | For resolving forces vertically (3 terms needed) | M1 | |
| | | $N + 25 \sin 15^\circ = 3 \times 10$ (√ cos instead of sin following sin instead of cos in (i)) | A1 √ | |
| | | Component is 23.5 N | A1 | 3 |
| 4 | (i) | KE (gain) = $\frac{1}{2}0.15 \times 8^2$ | B1 | |
| | | For using PE loss = KE gain | M1 | |
| | | Height is 3.2 m | A1 | 3 |
| | (ii) | For using WD is difference in PE loss and KE gain | M1 | |
| | | $WD = 0.15 \times 10 \times 4 - \frac{1}{2}0.15 \times 6^2$ | A1 | |
| | | Work Done is 3.3 J | A1 | 3 |

SR For candidates who treat *AB* as if it is straight and vertical (implicitly or otherwise) Max 2 out of 6 marks.

(i) $s = 8^2 \div (2 \times 10) = 3.2$ B1

(ii) $a = 6^2 \div (2 \times 4) = 4.5$ and $R = 0.15 \times 10 - 0.15 \times 4.5 = 0.825$ and
WD = $4 \times 0.825 = 3.3$ B1

| Page 2 | Mark Scheme | Syllabus | Paper |
|--------|---------------------------------------|-------------|----------|
| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 4 |

| | | | | |
|----------|-------|--|------------------|---|
| 5 | (i) | For applying Newton's second law to <i>A</i> or to <i>B</i> (3 terms needed) | M1 | |
| | | $T - 0.6 = 0.4a$ or $0.1g - T = 0.1a$ | A1 | |
| | | For a second of the above 2 equations or for $0.1g - 0.6 = 0.5a$ [Can be scored in part (ii)] (Sign of <i>a</i> must be consistent with that in first equation) | B1 | |
| | | Tension is 0.92 N | A1 | 4 |
| | (ii) | $a = 0.8$ | B1 | |
| | | For using $v = at$ | M1 | |
| | | Speed = 1.2 ms^{-1} | A1 | 3 |
| 6 | (i) | $T_{BM} = T_{AM}$ or $T_{BM}\cos 30^\circ = T_{AM}\cos 30^\circ$ | B1 | |
| | | For resolving forces at <i>M</i> horizontally ($2T \sin 30^\circ = 5$) or for using the sine rule in the triangle of forces ($T \div \sin 60^\circ = 5 \div \sin 60^\circ$) or for using Lami's theorem ($T \div \sin 120^\circ = 5 \div \sin 120^\circ$) | M1 | |
| | | Tension is 5 N A.G. | A1 | 3 |
| | (ii) | For resolving forces on <i>B</i> horizontally ($N = T \sin 30^\circ$) or from symmetry ($N = 5/2$) or for using Lami's theorem ($N \div \sin 150^\circ = 5 \div \sin 90^\circ$) | M1 | |
| | | For resolving forces on <i>B</i> vertically (3 terms needed) or for using Lami's theorem | M1 | |
| | | $0.2 \times 10 + F = T \cos 30^\circ$ or ($0.2g + F$) $\div \sin 120^\circ = T \div \sin 90^\circ$ | A1 | |
| | | For using $F = \mu R$ ($2.33 = 2.5\mu$) | M1 | |
| | | Coefficient is 0.932 | A1 | 5 |
| | (iii) | $(0.2 + m)g - 2.33 = 5 \cos 30^\circ$ or $mg = 2(2.33)$ $m = 0.466$ | B1 \sqrt B1 | 2 |
| 7 | (i) | For using the idea that area represents the distance travelled. | M1 | |
| | | For any two of $\frac{1}{2} \times 100 \times 4.8$, $\frac{1}{2} \times 200(4.8 + 7.2)$, $\frac{1}{2} \times 200 \times 7.2$ (240, 1200, 720) | A1 | |
| | | Distance is 2160 m | A1 | 3 |

| Page 3 | Mark Scheme | Syllabus | Paper |
|--------|--------------------------------|----------|-------|
| | A AND AS LEVEL – NOVEMBER 2003 | 9709 | 4 |

- (ii) For using the idea that the initial acceleration is the gradient of the first line segment or for using $v = at$ ($4.8 = 100a$) or $v^2 = 2as$ ($4.8^2 = 2a \times 240$) M1
 Acceleration is 0.048 ms^{-2} A1 2
- (iii) $a = 0.06 - 0.00024t$ B1
 Acceleration is greater by 0.012 ms^{-2} [\checkmark for $0.06 - \text{ans(ii)}$ (must be +ve) and/or wrong coefficient of t in $a(t)$] B1 \checkmark 2
 [Accept 'acceleration is 1.25 times greater']
- (iv) B 's velocity is a maximum when $0.06 - 0.00024t = 0$ B1 \checkmark
 [\checkmark wrong coefficient of t in $a(t)$]
 For the method of finding the area representing s_A (250) M1
 $240 + \frac{1}{2}(4.8 + 6.6)150$ or
 $240 + (4.8 \times 150 + \frac{1}{2} 0.012 \times 150^2)$ (1095) A1
 For using the idea that s_B is obtained from integration M1
 $0.03t^2 - 0.00004t^3$ A1
 Required distance is 155 m A1 \checkmark 6
 (\checkmark dependent on both M marks)

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GCE A AND AS LEVEL

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| MARK SCHEME |
| MAXIMUM MARK: 50 |
| SYLLABUS/COMPONENT: 9709/05, 8719/05 MATHEMATICS AND HIGHER MATHEMATICS Paper 5 (Mechanics 2) |

| Page 1 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 5 |

| | | |
|----------|--|------------|
| 1 | For using Newton's second law with $a = v^2/r$ | M1 |
| | $F = 50\,000 \frac{25^2}{1250}$ | A1 |
| | Magnitude of the force is 25 000 N | A1 |
| | | [3] |
| 2 | (i) For stating or implying that the centre of mass is vertically above the lowest point of the cone, and with $\bar{y} = 5$ | B1 |
| | For using $\tan \theta = \frac{10}{y}$ or equivalent | M1 |
| | $\theta = 63.4^\circ$ | A1 |
| | | [3] |
| (ii) | For using $F < \mu R$ | M1 |
| | $mg \sin \theta < \mu mg \cos \theta$ | A1 |
| | Alternative for the above 2 marks: | |
| | For using $\mu = \tan \phi$ where ϕ is the angle of friction | M1 |
| | $\phi > \theta$ because cone topples without sliding | A1 |
| | Coefficient is greater than 2 (ft on $\tan \theta$ in (i)) | A1ft |
| | N.B. Direct quotation of "topples if $\mu > \tan \theta$ " (scores B2); $\mu > 2$ (B1) | [3] |
| 3 | (i) $T = \frac{88 \times 0.1}{0.4}$ | B1 |
| | For using Newton's second law ($22 - 0.2 \times 10 = 0.2a$) (3 term equation needed) | M1 |
| | Initial acceleration is 100 ms^{-2} | A1 |
| | | [3] |
| (ii) | For using $\text{EPE} = \frac{\lambda x^2}{2L}$ $(\frac{88 \times 0.1^2}{2 \times 0.4})$ | M1 |
| | Initial elastic energy is 1.1 J | A1 |
| | | [2] |
| (iii) | Change in GPE = $0.2 \times 10 \times 0.1$ | B1 |
| | For using the principle of conservation of energy (KE, EPE and GPE must all be represented) | M1 |
| | $[\frac{1}{2} 0.2v^2 = 1.1 - 0.2]$ | |
| | Speed is 3 ms^{-1} | A1 |
| | | [3] |

| Page 2 | Mark Scheme | Syllabus | Paper |
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| | A AND AS LEVEL – NOVEMBER 2003 | 9709/8719 | 5 |

- 4 (i) e.g. For taking moments about BC M1
- Distance of centre of mass of triangular portion is
- $$9.5 + \frac{1}{3} \times 6 (= 11.5) \quad B1$$
- $$8 \times 9.5 \times 4.75 + \frac{1}{2} \times 8 \times 6 \times 11.5 = (8 \times 9.5 + \frac{1}{2} \times 8 \times 6) \bar{x} \quad A1ft$$
- Distance is 6.37 cm A1
- N.B. Alternative method
- e.g. Moments about axis through A perpendicular to AB M1
- Distance of C.O.M. of triangular piece removed is 2 B1
- $$(8 \times 15.5) \times 7.75 - (\frac{1}{2} \times 8 \times 6) \times 2 = (124 - 20) \bar{x}_1 \quad A1ft$$
- $(\bar{x}_1 = 9.13)$ therefore distance is 6.37 cm A1
- [4]
- (ii) For taking moments about A M1
- For LHS of $80(15.5 - 6.37) = T \times 15.5 \sin 30^\circ$ A1ft
- For RHS of above equation A1
- Tension is 94.2 N A1
- [4]
- (iii) For resolving forces on the lamina vertically (3 term equation) M1
- $(V = 80 - 94.2 \times 0.5)$ or taking moments about B
- $(15.5V = 8 \times 10 \times 6.37)$
- Magnitude of vertical component is 32.9 N A1ft
- [2]

| Page 3 | Mark Scheme | Syllabus | Paper |
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- 5 (i) For using $\dot{y} = \dot{y}_0 - gt$ with $\dot{y} = 0$ ($t = 2\sin\alpha$) M1
- For using $y = \dot{y}_0 t - \frac{1}{2}gt^2$ with t as found and $y = 7.2$, or show M1
- $t = 1.2$ as in (ii)
- Alternatively for using $y_{max} = \frac{V^2 \sin^2 \alpha}{2g}$ with $y_{max} = 7.2$ and $V = 20$
- or $\dot{y}^2 = \dot{y}_0^2 - 2gy$ with $\dot{y} = 0$ M2
- $7.2 = \frac{400 \sin^2 \alpha}{20}$ A1
- Angle is 36.9° A1
- [4]
- (ii) Speed on hitting the wall is 20×0.8 B1ft
(use of ball rebounding at 10 ms^{-1} scores B0)
- For using $y = 0 - \frac{1}{2}gt^2$ ($-7.2 = -\frac{1}{2}10t^2$) or
- $0 = \dot{y} - gt$ ($0 = 12 - 10t$) M1
- $t = 1.2$ A1
- Distance is 9.6 m (No ft if rebound velocity = 10 ms^{-1}) A1ft
- Alternative** – speed on hitting the wall is 20×0.8 B1ft
Use trajectory equation, with $\theta = 0^\circ$ M1
- $-7.2 = x \tan 0^\circ - \frac{gx^2}{2.8^2 \cos^2 0^\circ}$ (allow ft with halving attempt including 10) A1ft
- $x = 9.6 \text{ m}$ A1
- [4]
- (iii) $\dot{y} = \mp 10 \times 1.2$ B1ft
- $\theta = \tan^{-1}(\mp) \frac{\dot{y}}{\dot{x}}$ (\dot{x} must have halving attempt. Allow $\dot{x} = 10$) M1
- Required angle is 56.3° A1
- [3]

| Page 4 | Mark Scheme | Syllabus | Paper |
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- 6 (i) For using Newton's second law M1
- $$120 - 8v - 80 \times 10 \times 0.1 = 80a \quad \text{A1}$$
- $$\frac{1}{5-v} \frac{dv}{dt} = \frac{1}{10} \text{ from correct working} \quad \text{A1}$$
- [3]**
- (ii) For separating the variables and attempting to integrate M1
- $$-\ln(5-v) = \frac{1}{10}t + (C) \quad \text{A1}$$
- For using $v(0) = 0$ to find C (or equivalent by using limits) M1
($C = -\ln 5$)
- For converting the equation from logarithmic to exponential form M1
(allow even if $+ C$ omitted) ($5 \div (5-v) = e^{t/10}$)
- $$v = 5(1 - e^{-t/10}) \text{ from correct working} \quad \text{A1}$$
- [5]**
- (iii) For using $v = \frac{dx}{dt}$ and attempting to integrate M1
- $$x = 5(t + 10e^{-t/10}) + (C) \quad \text{A1ft}$$
- For using $x(0) = 0$ to find $(C) (= -50)$, then substituting $t = 20$ M1
(or equivalent using limits)
- Length is 56.8 m A1
- OR**
- For using Newton's second law with $a = v \frac{dv}{dx}$, separating the variables and M1
attempting to integrate
- $$-v - 5\ln(5-v) = \frac{x}{10} + C \quad \text{A1}$$
- For using $v = 0$ when $x = 0$ to find $C (= -5\ln 5)$, then substituting M1
 $t = 20$ into $v(t)$
- $$(v(20) = 5(1 - e^{-2}) = 4.3233),$$
- And finally substituting $v(20)$ into the above equation
- $$(x = -50(1 - e^{-2}) + 50 \times 2 = 50 + 50e^{-2}) \quad \text{M1}$$
- Length is 56.8m A1
- [4]**

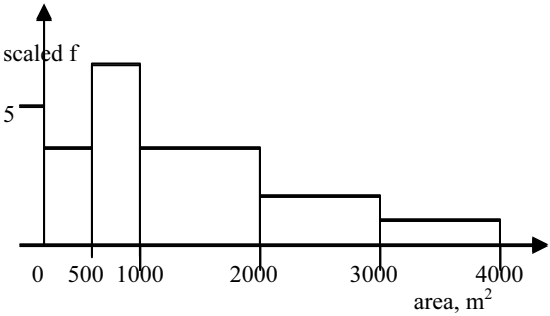
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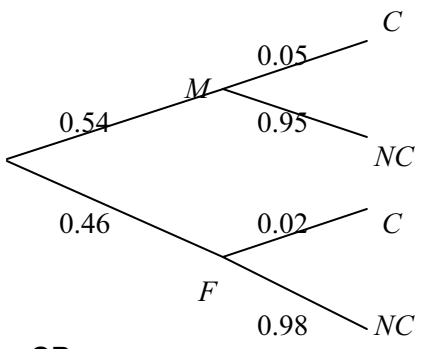
**GCE A AND AS LEVEL
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| MARK SCHEME |
| MAXIMUM MARK: 50 |
| SYLLABUS/COMPONENT: 9709/06, 0390/06 MATHEMATICS Paper 6 (Probability and Statistics 1) |

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|--------|---|-----------|-------|
| Page 1 | Mark Scheme | Syllabus | Paper |
| | AICE AND A AND AS LEVEL – NOVEMBER 2003 | 9709/0390 | 6 |

| | | |
|---|---|--|
| <p>1</p> <p>x 0 2 freq 23 17</p> <p>OR</p> <p>$P(0) = 23/40, P(2) = 17/40$ Mean = $34/40 = 0.850$ Variance = $(4 \times 17) / 40 - (0.85)^2$ = 0.978 (exact answer 0.9775) (391/400)</p> | <p>M1</p> <p>A1 M1 A1ft 4</p> | <p>For reasonable attempt at the mean using freq or probs but not using prob=0.5</p> <p>For correct mean For correct variance formula For correct answer</p> |
| <p>frequencies: 3, 7, 6, 3, 1 scaled frequencies: 3, 7, 3, 1.5, 0.5 or 0.006, 0.014, 0.006, 0.003, 0.001</p>  | <p>M1</p> <p>A1</p> <p>B1</p> <p>B1 4</p> | <p>For frequencies and attempt at scaling, accept cw/freq but not cw × freq, not cw/mid point</p> <p>For correct heights from their scaled frequencies seen on the graph</p> <p>For correct widths of bars, uniform horiz scale, no halves or gaps or less-than-or-equal tos</p> <p>Both axes labelled, fd and area or m². Not class width</p> |
| <p>3 $28 - \mu = 0.496\sigma$ (accept 0.495 or in between)</p> <p>$35 - \mu = 1.282\sigma$ (accept 1.281 or in between, but not 1.28)</p> <p>$\sigma = 8.91$ (accept 8.89 to 8.92 incl) $\mu = 23.6$</p> | <p>M1</p> <p>A1 A1</p> <p>M1</p> <p>A1 A1 6</p> | <p>For any equation with μ and σ and a reasonable z value not a prob. Allow cc, $\sqrt{\sigma}$, σ^2, or – and give M1 A0A1ft for these four cases</p> <p>For 2 correct equations</p> <p>For solving their two equations by elim 1 variable sensibly</p> <p>For correct answer For correct answer</p> |
| <p>4 (i) $(0.95)^5$ = 0.774</p> <p>(ii) $(0.95)^4 \times (0.05)^1 \times {}_5C_1$ = 0.204</p> <p>(iii) $(0.95)^2 \times (0.05)$ = 0.0451(361/8000)</p> | <p>M1</p> <p>A1 2</p> <p>M1</p> <p>A1 2</p> <p>M1</p> <p>A1 2</p> | <p>For 0.95 seen, can be implied For correct final answer</p> <p>For any binomial calculation with 3 terms, powers summing to 5</p> <p>For correct answer</p> <p>For no Ps, no Cs, and only 3 terms of type $p^2(1-p)$ For correct answer</p> |

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| <p>5</p>  <p>OR</p> $P(M C) = \frac{0.54 \times 0.05}{0.54 \times 0.05 + 0.46 \times 0.02}$ $= 0.746 \text{ (135/181)}$ | <p>M1 A1</p> <p>M1 A1</p> <p>M1 B1 M1 A1</p> <p>6</p> | <p>For correct shape ie <i>M</i> and <i>F</i> first</p> <p>All correct, ie labels and probabilities, no labels gets M1 only for (implied)correct shape</p> <p>For finding $P(M \text{ and } C)$ and $P(F \text{ and } C)$ For using 4 correct probs</p> <p>For correct conditional probability For correct numerator For summing two two-factor 'terms' For correct answer</p> |
| <p>6 (a) (i) 18564 (ii) ${}_{17}C_5$ or $6/18 \times$ their (i) or ${}_{18}C_6 - {}_{17}C_6$ = 6188</p> <p>(b) (i) 40320 (ii) $5! \times 3! \times {}_4C_1$ = 2880</p> | <p>B1 1 M1 A1 2</p> <p>B1 1 B1 B1 B1 4</p> | <p>For correct final answer For using 17 and 5 as a perm or comb For correct answer</p> <p>For correct final answer For $5!$ or ${}_3P_5$ used in a prod or quotient with a term $\neq 5!$ For $3!$ For ${}_4C_1$, may be implied by $4!$ For correct final answer</p> |
| <p>7 (i) $z = \pm 1.143$ $P(7.8 < T < 11) = \Phi(1.143) - 0.5$ = $0.8735 - 0.5$ = 0.3735 (accept ans rounded to 0.37 to 0.374)</p> <p>(ii) $(0.1265)^2 \times (0.8735) \times {}_3C_2$ = 0.0419</p> <p>(iii) Not symmetric so not normal Does not agree with the hospital's figures</p> | <p>M1 A1 M1 A1 4</p> <p>M1 A1ft 2</p> <p>B1 B1dep 2</p> | <p>For standardising, can be implied, no cc, no σ^2 but accept $\sqrt{\sigma}$ For seeing 0.8735 For subtracting two probs, $p_2 - p_1$ where $p_2 > p_1$ For correct answer</p> <p>For any three term binomial-type expression with powers summing to 3 For correct answer ft on their 0.8735/0.1265</p> <p>For any valid reason For stating it does not agree, with no invalid reasons</p> |
| <p>8 (i) $18c = 1$ $c = 1/18 = 0.0556$</p> <p>(ii) $E(X) = 2.78$ (= 25/9) (= 50c) $\text{Var}(X) = 1.17$ (= 95/81) (= 160c - 2500 c²)</p> <p>(iii) $P(X > 2.78) = 11c$ = 0.611 (= 11/18)</p> | <p>M1 A1 2</p> <p>M1 A1ft M1 A1ft 4</p> <p>M1 A1 2</p> | <p>For $\sum p_i = 1$ For correct answer</p> <p>Using correct formula for $E(X)$ For correct expectation, ft on their c For correct variance formula For correct answer ft on their c</p> <p>For using their correct number of discrete values of X For correct answer</p> |

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

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| MARK SCHEME |
| MAXIMUM MARK: 50 |
| SYLLABUS/COMPONENT: 9709/07, 8719/07 MATHEMATICS AND HIGHER MATHEMATICS Paper 7 (Probability and Statistics 2) |

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| <p>1 $\frac{1.9}{\sqrt{n}} \times 1.96 < 1$ $n > 13.9$ (13.87) $n = 14$</p> | <p>M1 A1 M1 A1 [4]</p> | <p>For equality or inequality involving width or equivalent and term in $1/\sqrt{n}$ and a z-value For correct inequality For solving a relevant equation For correct answer two</p> |
| <p>2 $\lambda = 4.5$ $P(X = 2, 3, 4) = e^{-4.5} \left(\frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!} \right)$ $= 0.471$</p> | <p>M1 B1 M1 A1 A1 [5]</p> | <p>For using Poisson approximation any mean For correct mean used For calculating P(2, 3, 4) their mean For correct numerical expression For correct answer NB Use of Normal can score B1 M1 SR Correct Bin scores M1 A1 A1 only</p> |
| <p>3 $SU \sim N(19,12)$ $P(T - SU > 0) \text{ or } P(T - S > 5) = 1 - \Phi\left(\frac{0-1}{\sqrt{21}}\right)$ $= \Phi(0.2182)$ $= 0.586$</p> | <p>B1 M1 M1 M1 A1 [5]</p> | <p>For correct mean and variance. Can be implied if using P(T-S>5) in next part For consideration of P(T - SU > 0) For summing their two variances For normalising and finding correct area from their values For correct answer</p> |
| <p>4 (i) $\lambda = \frac{20}{80} = 0.25$ $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - e^{-0.25} \left(1 + 0.25 + \frac{0.25^2}{2} \right)$ $= 0.00216$ (ii) $e^{\frac{-k}{80}} = 0.9$ $\frac{-k}{80} = -0.10536$ $k = 8.43$</p> | <p>B1 M1 M1 A1 [4] M1 M1 M1 A1 [4]</p> | <p>For $\lambda = 0.25$ For calculating a relevant Poisson prob(any λ) For calculating expression for P($X \geq 3$) their λ For correct answer For using $\lambda = -t/80$ in an expression for P(0) For equating their expression to 0.9 For solving the associated equation For correct answer two</p> |
| <p>5 (i) $P(\bar{X} > 1800) = 1 - \Phi\left(\frac{1800 - 1850}{117/\sqrt{26}}\right)$ $= \Phi(2.179)$ $= 0.985$</p> | <p>B1 M1 A1 [3]</p> | <p>For $117/\sqrt{26}$ (or equiv) For standardising and use of tables For correct answer two</p> |

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| <p>(ii) $H_0: \mu = 1850$ $H_1: \mu \neq 1850$</p> $\text{Test statistic} = \frac{1833 - 1850}{117/\sqrt{26}}$ $= -0.7409$ <p>Critical value $z = \pm 1.645$</p> <p>Accept H_0, no significant change</p> | <p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1ft</p> <p>[5]</p> | <p>Both hypotheses correct</p> <p>Standardising attempt including standard error</p> <p>Correct test statistic (+/-)</p> <p>Comparing with $z = \pm 1.645$, + with + or - with - (or equiv area comparison) ft 1 tail test $z=1.282$</p> <p>For correct conclusion on their test statistic and their z. No contradictions.</p> |
| <p>6 (i) (a) Rejecting H_0 when it is true (b) Accepting H_0 when it is false</p> <p>(ii) (a) $P(\text{NNNNN})$ under $H_0 = (0.94)^5$ $= 0.7339$ $P(\text{Type I error}) = 1 - 0.7339$ $= 0.266$</p> <p>(b) $P(\text{NNNNN})$ under $H_1 = (0.7)^5$ $= 0.168$ $P(\text{Type II error}) = 0.168$</p> | <p>B1</p> <p>B1</p> <p>[2]</p> <p>M1*</p> <p>A1</p> <p>M1*</p> <p>A1ft</p> <p>dep*</p> <p>[4]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> | <p>Or equivalent</p> <p>For evaluating $P(\text{NNNNN})$ under H_0</p> <p>For correct answer (could be implied)</p> <p>For identifying the Type I error outcome</p> <p>For correct final answer</p> <p>SR If M0M0 allow B1 for Bin(5,0.94)used</p> <p>For Bin(5,0.7) used</p> <p>For $P(\text{NNNNN})$ under H_1</p> <p>For correct final answer</p> |
| <p>7 (i) $\int_0^{\infty} ke^{-3x} dx = 1$</p> $0 - \frac{-k}{3} = 1 \Rightarrow k = 3$ <p>(ii) $\int_0^{q_1} 3e^{-3x} dx = 0.25$</p> $\left[-e^{-3x} \right]_0^{q_1} = 0.25$ $-e^{-3q_1} + 1 = 0.25$ $0.75 = e^{-3q_1}$ $q_1 = 0.0959$ | <p>M1</p> <p>A1</p> <p>[2]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p> | <p>For attempting to integrate from 0 to ∞ and putting the integral = 1</p> <p>For obtaining given answer correctly</p> <p>For equating $\int 3e^{-3x} dx$ to 0.25 (no limits needed)</p> <p>For attempting to integrate and substituting (sensible) limits and rearranging</p> <p>For correct answer</p> |

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| <p>(iii) Mean = $\int_0^{\infty} 3xe^{-3x} dx$</p> $= \left[-xe^{-3x} \right]_0^{\infty} - \int_0^{\infty} -e^{-3x} dx$ $= \left[\frac{e^{-3x}}{-3} \right]_0^{\infty}$ $= 0.333 \text{ or } 1/3$ | <p>B1 M1 A1 M1 A1 A1 [6]</p> | <p>For correct statement for mean For attempting to integrate $3xe^{-3x}$ (no limits needed) For $-xe^{-3x}$ or $-xe^{-3x}/3$ For attempt $\int -e^{-3x} dx$ (their integral) For $0+ \left[\frac{e^{-3x}}{-3} \right]_0^{\infty}$ For correct answer</p> |
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