CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values -2 and $\frac{1}{4}$	**A1**	
	State final answer $-2 < x < \frac{1}{4}$	**A1**	
	OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**	
	Obtain critical value $x = \frac{1}{4}$ similarly	**B2**	
	State final answer $-2 < x < \frac{1}{4}$	**B1**	[4]
	[Do not condone $\leqslant$ for $<$]		
2	State or imply $1 + u = u^2$	**B1**	
	Solve for u	**M1**	
	Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
	Use correct method for finding x from a positive root	**M1**	
	Obtain $x = 0.438$ and no other answer	**A1**	[5]
3	Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$	**M1***	
	Substitute throughout for $\tan\theta$ or for $\tan\phi$	**dep M1***	
	Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic and find an angle	**M1**	
	Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
	Obtain answer $\theta = 53.1°$, $\phi = 161.6°$	**A1**	[6]
	[Treat answers in radians as a misread. Ignore answers outside the given interval.]		
	[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]		
4 (i)	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent	**M1**	
	Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
(ii)	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
	Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 2.219	**A1**	
	Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

5	(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	B1	
		Use product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Use Pythagoras	M1	
		Justify the given form	A1	[5]
	(ii)	Fully justify the given statement	B1	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	B1	[1]
6	(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	B1	
		Substitute $x = -\frac{1}{2}$ and equate the result to 1	M1	
		Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	A1	
		Solve for a or for b	M1	
		Obtain $a = 6$ and $b = -3$	A1	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	M1	
		Obtain quadratic factor $8x^2 - 2x - 1$	A1	
		Obtain factorisation $(x+1)(4x+1)(2x-1)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B.] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]		
7	(i)	Use correct method to form a vector equation for AB	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
	(ii)	Using a direction vector for AB and a relevant point, obtain an equation for m in any form	M1	
		Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
	(iii)	Express general point of AB in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
		Substitute in equation of m and solve for λ or for μ	M1	
		Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of N, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
		Carry out a correct method for finding CN	M1	
		Obtain the given answer $\sqrt{13}$	A1	[5]
		[The f.t. is on the direction vector for AB.]		

8		Separate variables and integrate one side	B1	
		Obtain term $\ln(x+2)$	B1	
		Use cos $2A$ formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	M1	
		Obtain correct form $(1-\cos 4\theta)/2$, or equivalent	A1	
		Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	A1√	
		Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	M1	
		Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	A1	
		Use correct method for solving an equation of the form $\ln(x+2) = f$	M1	
		Obtain answer $x = 0.962$	A1	[9]
9	(i)	Show u in a relatively correct position	B1	
		Show u^* in a relatively correct position	B1	
		Show $u^* - u$ in a relatively correct position	B1	
		State or imply that $OABC$ is a parallelogram	B1	[4]
	(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent	M1	
		Simplify the numerator to 8 + 6i or the denominator to 10	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	[3]
	(iii)	State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	B1	
		Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	M1	
		Fully justify the given statement using exact values	A1	[3]
10	(i)	Use the quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	A1	[4]
	(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	M1	
		State indefinite integral $\frac{1}{3}\ln(1+x^3)$	A1	
		Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	M1	
		State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	A1	
		Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	M1	
		Obtain answer $p = 3.40$	A1	[2]