UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2010 question paper for the guidance of teachers

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 Obtain $1 - 6x$ B1
State correct unsimplified x^2 term. Binomial coefficients must be expanded. M1
Obtain … $+ 24x^2$ A1 [3]

2 Use of correct quotient or product rule to differentiate x or t M1
Obtain correct $\dfrac{3}{(2t+3)^2}$ or unsimplified equivalent A1
Obtain $-2e^{-2t}$ for derivative of y B1
Use $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$ or equivalent M1
Obtain –6 cwo A1 [5]

Alternative:
Eliminate parameter and attempt differentiation $\left(y = e^{\frac{-6x}{1-2x}} \right)$ B1
Use correct quotient or product rule M1
Use chain rule M1
Obtain $\dfrac{dy}{dx} = \dfrac{-6}{(1-2x)^2} e^{\frac{-6x}{1-2x}}$ A1
Obtain –6 cwo A1

3 **(i)** Attempt multiplication and use $i^2 = -1$ M1
Obtain $3 + 4i$ A1
Obtain 5 for <u>modulus</u> B1 [3]

(ii) Draw complete circle with centre corresponding to their w^2 … B1√
… and radius corresponding to their $|w^2|$ B1√
Shade the correct region cwo B1 [3]

4 **(i)** Obtain derivative of form $k\cos 3x \sin 3x$, any constant k M1
Obtain $-24\cos 3x \sin 3x$ or unsimplified equivalent A1
Obtain $-6\sqrt{3}$ or exact equivalent A1 [3]

(ii) Express integrand in the form $a + b\cos 6x$, where $ab \neq 0$ M1
Obtain $2 + 2\cos 6x$ o.e. A1
Obtain $2x + \frac{1}{3}\sin 6x$ or equivalent, condoning absence of $+ c$, ft on a, b A1√ [3]

5 State or imply form $\frac{A}{2x+1}+\frac{B}{x+2}$ B1

Use relevant method to find A or B M1

Obtain $\frac{4}{2x+1}-\frac{1}{x+2}$ A1

Integrate and obtain $2\ln(2x+1)-\ln(x+2)$ (ft on their A, B) B1√B1√

Apply limits to integral containing terms $a\ln(2x+1)$ and $b\ln(x+2)$ and apply a law of logarithms correctly. M1

Obtain given answer $\ln 50$ correctly A1 [7]

6 **(i)** State general vector for point on line, e.g.
$-5\mathbf{i}+3\mathbf{j}+6\mathbf{k}+s(10\mathbf{i}+5\mathbf{j}-5\mathbf{k})$ or $5\mathbf{i}+8\mathbf{j}+\mathbf{k}+t(10\mathbf{i}+5\mathbf{j}-5\mathbf{k})$ or equiv B1

Substitute their line into equation of plane and solve for parameter M1

Obtain correct value, $s=\frac{2}{5}$ or $t=-\frac{3}{5}$ or equivalent A1

Obtain (–1, 5, 4) o.e. A1 [4]

(ii) State or imply normal vector to p is $2\mathbf{i}-\mathbf{j}+4\mathbf{k}$ B1

Carry out process for evaluating scalar product of two relevant vectors M1

Using correct process for moduli, divide scalar product by the product of the moduli and evaluate arcsin(..) or arccos(..) of the result. M1

Obtain 5.1° or 0.089 rads A1 [4]

7 **(i)** Attempt integration by parts M1

Obtain $-x^{-1}\ln x+\int\frac{1}{x^2}\,dx$, $\frac{x\ln x-x}{x^2}+2\int\frac{\ln x}{x^2}\,dx-2\int\frac{1}{x^2}\,dx$ or equivalent A1

Obtain $-x^{-1}\ln x-x^{-1}$ or equivalent A1

Use limits correctly, equate to $\frac{2}{5}$ and attempt rearrangement to obtain a in terms of $\ln a$ M1

Obtain given answer $a=\frac{5}{3}(1+\ln a)$ correctly A1 [5]

(ii) Use valid iterative formula correctly at least once M1

Obtain final answer 3.96 A1

Show sufficient iterations to › 4 dp to justify accuracy to 2 dp or show sign change in interval (3.955, 3.965) A1 [3]

[4 → 3.9772 → 3.9676 → 3.9636 → 3.9619]

SR: Use of $a_{n+1}=e^{\left(\frac{3}{5}a_n-1\right)}$ to obtain 0.50 also earns 3/3.

8 **(i)** Obtain or imply $R = 4$ B1
Use appropriate trigonometry to find α M1
Obtain $\alpha = 52.24$ or better from correct work A1 [3]

(ii) **(a)** State or imply $\theta - \alpha = \cos^{-1}(-4 \div R)$ M1
Obtain 232.2 or better A1 [2]

(b) Attempt at least one value using $\cos^{-1}(3 \div R)$ M1
Obtain one correct value e.g. ± 41.41° A1
Use $\frac{1}{2}\theta - \alpha = \cos^{-1}\left(\frac{3}{R}\right)$ to find θ M1
Obtain 21.7 A1 [4]

9 **(i)** State $\frac{\mathrm{d}A}{\mathrm{d}t} = k\sqrt{2A-5}$ B1 [1]

(ii) Separate variables correctly and attempt integration of each side M1
Obtain $(2A-5)^{\frac{1}{2}} = \ldots$ or equivalent A1
Obtain = kt or equivalent A1
Use $t = 0$ and $A = 7$ to find value of arbitrary constant M1
Obtain $C = 3$ or equivalent A1
Use $t = 10$ and $A = 27$ to find k M1
Obtain $k = 0.4$ or equivalent A1
Substitute $t = 20$ and values for C and k to find value of A M1
Obtain 63 cwo A1 [9]

10 **(i)** Attempt to solve for m the equation p(–2) = 0 or equivalent M1
Obtain $m = 6$ A1 [2]

Alternative:
Attempt p(z) ÷ (z + 2), equate a constant remainder to zero and solve for m. M1
Obtain $m = 6$ A1

(ii) **(a)** State $z = -2$ B1
Attempt to find quadratic factor by inspection, division, identity, … M1
Obtain $z^2 + 4z + 16$ A1
Use correct method to solve a 3-term quadratic equation M1
Obtain $-2 \pm 2\sqrt{3}\mathrm{i}$ or equivalent A1 [5]

(b) State or imply that square roots of answers from part **(ii)(a)** needed M1
Obtain $\pm\mathrm{i}\sqrt{2}$ A1
Attempt to find square root of a further root in the form $x + \mathrm{i}y$ or in polar form M1
Obtain $a^2 - b^2 = -2$ and $ab = (\pm)\sqrt{3}$ following their answer to part **(ii)(a)** A1√
Solve for a and b M1
Obtain $\pm(1 + \mathrm{i}\sqrt{3})$ and $\pm(1 - \mathrm{i}\sqrt{3})$ A1 [6]