



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level



MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3 (P3)

October/November 2009

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)



READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

- 1 Solve the equation

$$\ln(5 - x) = \ln 5 - \ln x,$$

giving your answers correct to 3 significant figures. [4]

- 2 The equation $x^3 - 8x - 13 = 0$ has one real root.

(i) Find the two consecutive integers between which this root lies. [2]

(ii) Use the iterative formula

$$x_{n+1} = (8x_n + 13)^{\frac{1}{3}}$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

- 3 The equation of a curve is $x^3 - x^2y - y^3 = 3$.

(i) Find $\frac{dy}{dx}$ in terms of x and y . [4]

(ii) Find the equation of the tangent to the curve at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$. [2]

- 4 The angles α and β lie in the interval $0^\circ < x < 180^\circ$, and are such that

$$\tan \alpha = 2 \tan \beta \quad \text{and} \quad \tan(\alpha + \beta) = 3.$$

Find the possible values of α and β . [6]

- 5 The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $p(x)$. The result of differentiating $p(x)$ with respect to x is denoted by $p'(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and of $p'(x)$.

(i) Find the values of a and b . [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

- 6 (i) Use the substitution $x = 2 \tan \theta$ to show that

$$\int_0^2 \frac{8}{(4 + x^2)^2} dx = \int_0^{\frac{1}{4}\pi} \cos^2 \theta d\theta. \quad [4]$$

(ii) Hence find the exact value of

$$\int_0^2 \frac{8}{(4 + x^2)^2} dx. \quad [4]$$

7 The complex numbers $-2 + i$ and $3 + i$ are denoted by u and v respectively.

(i) Find, in the form $x + iy$, the complex numbers

(a) $u + v$, [1]

(b) $\frac{u}{v}$, showing all your working. [3]

(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram with origin O , the points A , B and C represent the complex numbers u , v and $u + v$ respectively.

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

(iv) State fully the geometrical relationship between the line segments OA and BC . [2]

8 (i) Express $\frac{1+x}{(1-x)(2+x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1+x}{(1-x)(2+x^2)}$ in ascending powers of x , up to and including the term in x^2 . [5]

9 The temperature of a quantity of liquid at time t is θ . The liquid is cooling in an atmosphere whose temperature is constant and equal to A . The rate of decrease of θ is proportional to the temperature difference $(\theta - A)$. Thus θ and t satisfy the differential equation

$$\frac{d\theta}{dt} = -k(\theta - A),$$

where k is a positive constant.

(i) Find, in any form, the solution of this differential equation, given that $\theta = 4A$ when $t = 0$. [5]

(ii) Given also that $\theta = 3A$ when $t = 1$, show that $k = \ln \frac{3}{2}$. [1]

(iii) Find θ in terms of A when $t = 2$, expressing your answer in its simplest form. [3]

10 The plane p has equation $2x - 3y + 6z = 16$. The plane q is parallel to p and contains the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

(i) Find the equation of q , giving your answer in the form $ax + by + cz = d$. [2]

(ii) Calculate the perpendicular distance between p and q . [3]

(iii) The line l is parallel to the plane p and also parallel to the plane with equation $x - 2y + 2z = 5$. Given that l passes through the origin, find a vector equation for l . [5]

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