Cambridge
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## Cambridge International Examinations

Cambridge International Advanced Subsidiary Level

MATHEMATICS
9709/21
Paper 2 Pure Mathematics 2 (P2)

Additional Materials: Answer Booklet/Paper Graph Paper List of Formulae (MF9)

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Use logarithms to solve the equation

$$
5^{x+3}=7^{x-1}
$$

2 A curve has equation

$$
\begin{equation*}
y=\frac{3 x+1}{x-5} \tag{5}
\end{equation*}
$$

Find the coordinates of the points on the curve at which the gradient is -4 .

3 (i) Express $8 \sin \theta+15 \cos \theta$ in the form $R \sin (\theta+\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$. Give the value of $\alpha$ correct to 2 decimal places.
(ii) Hence solve the equation

$$
\begin{equation*}
8 \sin \theta+15 \cos \theta=6 \tag{4}
\end{equation*}
$$

for $0^{\circ} \leqslant \theta \leqslant 360^{\circ}$.

4 (i) By sketching a suitable pair of graphs, show that the equation

$$
\begin{equation*}
\ln x=4-\frac{1}{2} x \tag{2}
\end{equation*}
$$

has exactly one real root, $\alpha$.
(ii) Verify by calculation that $4.5<\alpha<5.0$.
(iii) Use the iterative formula $x_{n+1}=8-2 \ln x_{n}$ to find $\alpha$ correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

5 (a) Find $\int\left(\tan ^{2} x+\sin 2 x\right) d x$.
(b) Find the exact value of $\int_{0}^{1} 3 \mathrm{e}^{1-2 x} \mathrm{~d} x$.

6 (i) Find the quotient and remainder when

$$
\begin{equation*}
x^{4}+x^{3}+3 x^{2}+12 x+6 \tag{4}
\end{equation*}
$$

is divided by $\left(x^{2}-x+4\right)$.
(ii) It is given that, when

$$
\begin{equation*}
x^{4}+x^{3}+3 x^{2}+p x+q \tag{2}
\end{equation*}
$$

is divided by $\left(x^{2}-x+4\right)$, the remainder is zero. Find the values of the constants $p$ and $q$.
(iii) When $p$ and $q$ have these values, show that there is exactly one real value of $x$ satisfying the equation

$$
\begin{equation*}
x^{4}+x^{3}+3 x^{2}+p x+q=0 \tag{3}
\end{equation*}
$$

and state what that value is.


The parametric equations of a curve are

$$
x=6 \sin ^{2} t, \quad y=2 \sin 2 t+3 \cos 2 t,
$$

for $0 \leqslant t<\pi$. The curve crosses the $x$-axis at points $B$ and $D$ and the stationary points are $A$ and $C$, as shown in the diagram.
(i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{2}{3} \cot 2 t-1$.
(ii) Find the values of $t$ at $A$ and $C$, giving each answer correct to 3 decimal places.
(iii) Find the value of the gradient of the curve at $B$.

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