UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)** **October/November 2008**

1 hour 15 minutes

6986198037

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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International Examinations

[Turn over

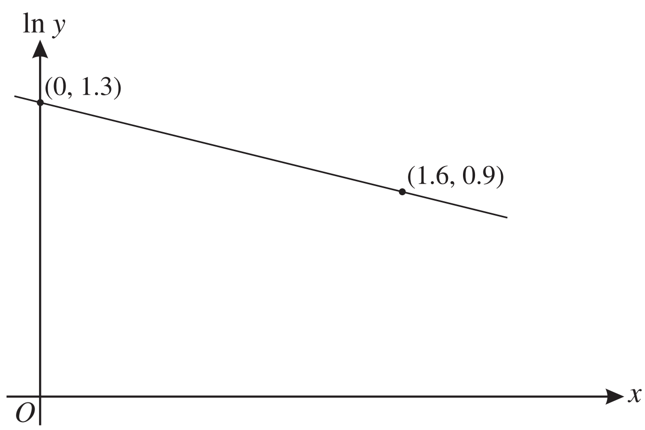
1 Solve the inequality $|x - 3| > |2x|$. [4]

2 The polynomial $2x^3 - x^2 + ax - 6$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, factorise $\mathrm{p}(x)$ completely. [3]

3



The variables x and y satisfy the equation $y = A(b^{-x})$, where A and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0, 1.3) and (1.6, 0.9), as shown in the diagram. Find the values of A and b, correct to 2 decimal places. [5]

4 **(i)** Show that the equation

$$\sin(x + 30^\circ) = 2\cos(x + 60^\circ)$$

can be written in the form

$$(3\sqrt{3})\sin x = \cos x.$$ [3]

(ii) Hence solve the equation

$$\sin(x + 30^\circ) = 2\cos(x + 60^\circ),$$

for $-180^\circ \leqslant x \leqslant 180^\circ$. [3]

5 Show that $\displaystyle\int_1^2 \left(\frac{1}{x} - \frac{4}{2x+1}\right) \mathrm{d}x = \ln \tfrac{18}{25}$. [6]

6 Find the exact coordinates of the point on the curve $y = x\mathrm{e}^{-\frac{1}{2}x}$ at which $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$. [7]

7 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cos x = 2 - 2x,$$

where x is in radians, has only one root for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = 1 - \tfrac{1}{2}\cos x_n$$

converges, then it converges to the root of the equation in part **(i)**. [1]

(iv) Use this iterative formula, with initial value $x_1 = 0.6$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(i)** **(a)** Prove the identity

$$\sec^2 x + \sec x \tan x \equiv \frac{1 + \sin x}{\cos^2 x}.$$

(b) Hence prove that

$$\sec^2 x + \sec x \tan x \equiv \frac{1}{1 - \sin x}.$$

[3]

(ii) By differentiating $\dfrac{1}{\cos x}$, show that if $y = \sec x$ then $\dfrac{dy}{dx} = \sec x \tan x$. [3]

(iii) Using the results of parts **(i)** and **(ii)**, find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{1}{1 - \sin x}\, dx.$$

[3]

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