Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	For a correctly selected term in $\frac{1}{x^2}$: $(3x)^4$ or 3^4	**B1**	Components of coefficient added together 0/4 B1 expect 81
	$\times\left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$	**B1**	B1 expect 8/27
	$\times\ {}_7C_3$ or ${}_7C_4$	**B1**	B1 expect 35
	$\rightarrow$ **840 or** $\boldsymbol{\frac{840}{x^2}}$	**B1**	All of the first three marks can be scored if the correct term is seen in an expansion **and it is selected** but then wrongly simplified.
			SC: A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2.
		4	

Question	Answer	Marks	Guidance
2	Integrate $\rightarrow \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C)	B1 B1	B1 for each term correct – allow unsimplified. C not required.
	$\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \rightarrow \dfrac{40}{3} - \dfrac{14}{3}$	M1	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
	$= \dfrac{26}{3}$ **oe**	A1	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
		4	

Question	Answer	Marks	Guidance
3(i)	P is $(t, 5t)$ Q is $(t, t(9-t^2)) \rightarrow 4t - t^3$	B1 B1	B1 for both y coordinates which can be implied by subsequent working. B1 for PQ allow $\lvert 4t - t^3 \rvert$ or $\lvert t^3 - 4t \rvert$. **Note:** $4x - x^3$ from equating line and curve 0/2 even if x then replaced by t.
		[2]	

Question	Answer	Marks	Guidance
3(ii)	$\frac{\mathrm{d}(PQ)}{\mathrm{d}t} = 4 - 3t^2$	**B1FT**	B1FT for differentiation of their *PQ*, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from **(i)** but not the equation of the curve.
	$= 0 \rightarrow t = + \frac{2}{\sqrt{3}}$	**M1**	Setting their differential of *PQ* to 0 and attempt to solve for t or *x*.
	$\rightarrow$ **Maximum** $\boldsymbol{PQ} = \frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$	**A1**	Allow 3.08 awrt. If answer comes from wrong method in **(i)** award A0. Correct answer from correct expression by T&I scores 3/3.
		3	

Question	Answer	Marks	Guidance
4(i)	$fg(x) = 2 - 3\cos(\frac{1}{2}x)$	**B1**	Correct *fg*
	$2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\,\frac{1}{3}\right)$	**M1**	M1 for correct order of operations to solve their *fg*(x) = 1 as far as using inverse cos expect 1.23, (or 70.5°) condone *x* =.
	$\boldsymbol{x} = $ **2.46 awrt or** $\frac{4.7\pi}{6}(0.784\pi\,\text{awrt})$	**A1**	One solution only in the given range, ignore answers outside the range. Answer in degrees A0.
			Alternative: Solve *f*(*y*) = 1 → y = 1.23→ $\frac{1}{2}x = 1.23$ **B1M1** → *x* = 2.46 **A1**
		3	

Question	Answer	Marks	Guidance
4(ii)		B1	One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted "V". If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark $0 \to 2\pi$.
		B1	Start and finish at roughly the same negative y value. Significantly more above the x axis than below or correct range implied by labels .
		B1	Fully correct. Curves not lines. Must be a reasonable curve clearly turning at both ends. Labels not required but must be appropriate if present.
		3	

Question	Answer	Marks	Guidance
5(i)	From the AP: $x-4=y-x$	B1	Or equivalent statement e.g. $y=2x-4$ or $x=\frac{y+4}{2}$.
	From the GP: $\frac{y}{x}=\frac{18}{y}$	B1	Or equivalent statement e.g. $y^2=18x$ or $x=\frac{y^2}{18}$.
	Simultaneous equations: $y^2-9y-36=0$ or $2x^2-17x+8=0$	M1	Elimination of either x or y to give a three term quadratic (= 0)
	OR		
	$4+d=x$, $4+2d=y \rightarrow \frac{4+2d}{4+d}=r$ oe	B1	
	$(4+d)\left(\frac{4+2d}{4+d}\right)^2=18 \rightarrow 2d^2-d-28=0$	M1	Uses $ar^2=18$ to give a three term quadratic (= 0)
	$d=4$	B1	Condone inclusion of $d=\frac{-7}{2}$ oe

Question	Answer	Marks	Guidance
5(i)	**OR**		
	From the GP $\frac{y}{x} = \frac{18}{y}$	**B1**	
	$\rightarrow x = \frac{y^2}{18} \rightarrow 4 + d = \frac{y^2}{18} \rightarrow d = \frac{y^2}{18} - 4$	**B1**	
	$4 + 2\left(\frac{y^2}{18} - 4\right) = y \rightarrow y^2 - 9y - 36 = 0$	**M1**	
	$\boldsymbol{x = 8, y = 12.}$	**A1**	Needs both x and y. Condone $\left(\frac{1}{2}, -3\right)$ included in final answer. Fully correct answer www 4/4.
		4	
5(ii)	AP 4th term = **16**	**B1**	Condone inclusion of $\frac{-13}{2}$ oe
	GP 4th term = $8 \times \left(\frac{12}{8}\right)^3$	**M1**	A valid method using their x and y from **(i)**.
	= **27**	**A1**	Condone inclusion of -108
			Note: Answers from fortuitous $x = 8$, $y = 12$ in **(i)** can only score M1. Unidentified correct answer(s) with no working seen after valid $x = 8$, $y = 12$ to be credited with appropriate marks.
		3	

Question	Answer	Marks	Guidance
6(i)	In Δ ABD, $\tan\theta = \frac{9}{BD} \rightarrow BD = \frac{9}{\tan\theta}$ or $9\tan(90-\theta)$ or $9\cot\theta$ or $\sqrt{\left[(20\tan\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule)	B1	Both marks can be gained for correct equated expressions.
	In Δ DBC, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$	B1	
	$20\sin\theta = \frac{9}{\tan\theta}$	M1	Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary.
	$\rightarrow 20\sin^2\theta = 9\cos\theta$ AG	A1	Correct manipulation of their expression to arrive at given answer.
			SC: In Δ DBC, $\sin\theta = \frac{BD}{20} \rightarrow BD = 20\sin\theta$ B1 In Δ ABD, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$ $\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \rightarrow \cos\theta = \frac{20\sin^2\theta}{9}$ M1 **$\rightarrow 20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4
		4	
6(ii)	Uses $s^2 + c^2 = 1 \rightarrow 20\cos^2\theta + 9\cos\theta - 20$ (= 0)	M1	Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$
	$\rightarrow \cos\theta = 0.8$	A1	www
	$\rightarrow \theta = 36.9°$ awrt	A1	www. Allow 0.644^c awrt. Ignore $323.1°$ or 2.50^c. Note: correct answer without working scores 0/3.
		3	

Question	Answer	Marks	Guidance
7	$\overrightarrow{PN} = 8\mathbf{i} - 8\mathbf{k}$	B1	
	$\overrightarrow{PM} = 4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$	B2,1,0	Loses 1 mark for each component incorrect
			SC: $\overrightarrow{PN} = -8\mathbf{i} + 8\mathbf{k}$ **and** $\overrightarrow{PM} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ scores 2/3.
	$\overrightarrow{PN}.\overrightarrow{PM} = 32 + 0 + 48 = 80$	M1	Evaluates $x_1x_2+y_1y_2+z_1z_2$ for correct vectors or one or both reversed.
	$\lvert PN\rvert \times \lvert PM\rvert = \sqrt{128} \times \sqrt{68}$ $(= 16\sqrt{34})$	M1	Product of their moduli – may be seen in cosine rule
	$\sqrt{128} \times \sqrt{68} \cos M\hat{P}N = 80$	M1	All linked correctly.
	Angle $M\hat{P}N =$ **31.0**$^\circ$ **awrt**	A1	Answer must come directly from +ve cosine ratio. Cosine rule not accepted as a complete method. Allow 0.540^c awrt. **Note:** Correct answer from incorrect vectors scores A0 (XP)
		7	

Question	Answer	Marks	Guidance
8(i)	$A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(¾)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ **or** $B\hat{A}C = \cos^{-1}(¾)$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$	**M1**	Correct method for $A\hat{B}C$, expect 1.696^cawrt Or for $B\hat{A}C$, expect 0.723^cawrt
	$C\hat{B}Y = \pi - A\hat{B}C$ or $2 \times C\hat{A}B$	**M1**	For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2 \times C\hat{A}B$
	OR		
	Find CY from $\Delta\, ACY$ using Pythagoras or similar Δs	**M1**	Expect $4\sqrt{7}$
	$C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(their\ CY)^2}{2\times8\times8}\right)$	**M1**	Correct use of cosine rule
	$\boldsymbol{C\hat{B}Y = 1.445^c}$ AG	**A1**	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
8(ii)	Arc $CY = 8 \times 1.445$	**B1**	Use of $s=8\theta$ for arc CY, Expect 11.56
	$B\hat{A}C = ½(\pi - A\hat{B}C)$ or $\cos^{-1}(¾)$	***M1**	For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect 0.7227^c
	Arc $XC = 12 \times$ (their $B\hat{A}C$)	**DM1**	Expect 8.673
	Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www**	**A1**	Omission of '+4' only penalised here.
		4	

Question	Answer	Marks	Guidance
9(i)	$2x^2 - 12x + 7 = 2(\boldsymbol{x}-3)^2 - 11$	**B1 B1**	Mark full expression if present: B1 for $2(x-3)^2$ and B1 for – 11. If no clear expression award $a = -3$ and $b = -11$.
		2	
9(ii)	Range (of f or y) ⩾ ‘their **– 11**’	**B1FT**	FT for their ‘b’ or start again. Condone **>.** Do **NOT** accept $x >$ or ⩾
		1	
9(iii)	(k =) **–**“their a” also allow ***x* or *k* ⩽ 3**	**B1FT**	FT for their “a” or start again using $\frac{dy}{dx} = 0$. Do **NOT accept *x* = 3.**
		1	
9(iv)	$y = 2(x-3)^2 - 11 \rightarrow y + 11 = 2(x-3)^2$ $\frac{y+11}{2} = (x-3)^2$	***M1**	Isolating their $(x-3)^2$, condone – 11.
	$x = 3 + \sqrt{\left(\frac{y+11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y+11}{2}\right)}$	**DM1**	Other operations in correct order, allow ± at this stage. Condone – 3.
	($g^{-1}(x)$ or y) $= 3 - \sqrt{\left(\frac{x+11}{2}\right)}$	**A1**	needs ‘–’. x and y could be interchanged at the start.
		3	

Question	Answer	Marks	Guidance
10(i)	$2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ → 3 term quadratic.	*M1	Attempt to eliminate y (or x) to form a 3 term quadratic. Expect $3x^2 - kx + 12$ or $3y^2 - 5ky + (2k^2 + 12)$ (= 0)
	Use of $b^2 - 4ac \to k^2 - 144 < 0$	DM1	Using the discriminant, allow $\leqslant$, = 0; expect 12 and −12
	$\mathbf{-12 < k < 12}$	A1	Do **NOT accept $\leqslant$.** Separate statements OK.
		3	
10(ii)	Using $k = 15$ in their 3 term quadratic	M1	From (i) or restart. Expect $3x^2 - 15x + 12$ or $3y^2 - 75y + 462$ (= 0)
	$x = 1,4$ or $y = 11, 14$	A1	Either pair of x or y values correct..
	(1, 14) and (4, 11)	A1	Both pairs of coordinates
		3	
10(iii)	Gradient of $AB = -1 \to$ Perpendicular gradient = +1	B1FT	Use of $m_1m_2 = -1$ to give +1 or ft from their A and B.
	Finding their midpoint using their (1, 14) and (4, 11)	M1	Expect (2½, 12½)
	Equation: $\mathbf{y - 12\frac{1}{2} = (x - 2\frac{1}{2})}$ [$y = x + 10$]	A1	Accept correct unsimplified and isw
		3	

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{3}{2}\times(4x+1)^{-\frac{1}{2}}\right]$ [×4] [− 2] $\left(\frac{6}{\sqrt{4x+1}}-2\right)$	**B2,1,0**	Looking for 3 components
	$\int y\,dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right]$ [÷ 4] [$-\frac{2x^2}{2}$] (+ C) $\left(=\frac{(4x+1)^{\frac{3}{2}}}{2}-x^2\right)$	**B1 B1 B1**	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '÷4'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating.
		5	
11(ii)	At M, $\frac{dy}{dx} = 0 \rightarrow \frac{6}{\sqrt{4x+1}} = 2$	**M1**	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as x = k)
	$\boldsymbol{x = 2, y = 5}$	**A1 A1**	
		3	

Question	Answer	Marks	Guidance
11(iii)	Area under the curve = $\left[\frac{1}{2}(4x+1)^{\frac{3}{2}} - x^2\right]_0^2$	M1	Uses their integral and their '2' and 0 correctly
	$(13.5 - 4) - 0.5$ or $9.5 - 0.5 = 9$	A1	No working implies use of integration function on calculator M0A0.
	Area under the chord = trapezium = ½ × 2 × (3 + 5) = 8 Or $\left[\frac{x^2}{2}+3x\right]_0^2 = 8$	M1	Either using the area of a trapezium with their 2, 3 and 5 or $\int(\textit{their } x+3)dx$ using their '2' and 0 correctly.
	(Shaded area = 9 – 8) **= 1**	A1	Dependent on both method marks,
	OR Area between the chord and the curve is:		
	$\int_0^2 3\sqrt{4x+1} - 2x - (x+3)dx$ $= \int_0^2 3\sqrt{4x+1} - 3x - 3dx$	M1	Subtracts their line from given curve and uses their '2' and 0 correctly.
	$= 3\left[\frac{1}{6}(4x+1)^{\frac{3}{2}} - \frac{x^2}{2} - x\right]_0^2$	A1	All integration correct and limits 2 and 0.
	$= 3\left\{\left(\frac{27}{6} - 2 - 2\right) - \left(\frac{1}{6}\right)\right\}$	M1	Evidence of substituting their '2' and 0 into their integral.
	$= 3\left\{\frac{1}{2} - \frac{1}{6}\right\} = 3\left\{\frac{1}{3}\right\} = 1$	A1	No working implies use of a calculator M0A0.
		[4]	