Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

* 8 1 0 3 9 3 5 2 1 7 *

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

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[Turn over

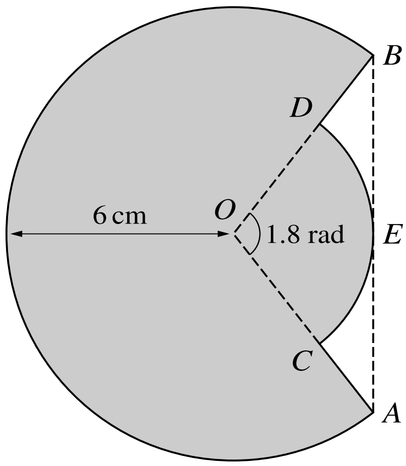
1 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

2 The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a. [4]

3 Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

5



The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle COD = 1.8 radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

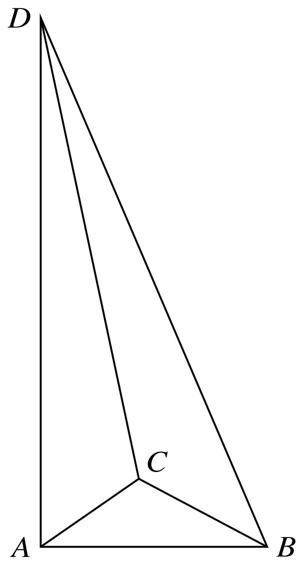
6 Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

7



The diagram shows a triangular pyramid $ABCD$. It is given that

$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB, DAC and CAB is 90°. [3]

(ii) Find the exact value of the area of the triangle ABC, and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

8 **(i)** Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $\mathrm{f}(x) = x^2 + 1$ and $\mathrm{fg}(x) = 4x^2 + 12x + 10$. Find $\mathrm{g}(x)$. [1]

(iii) Find $(\mathrm{fg})^{-1}(x)$ and give the domain of $(\mathrm{fg})^{-1}$. [4]

9 **(a)** Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

[Questions 10 and 11 are printed on the next page.]

10 A curve is such that $\frac{dy}{dx} = \frac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

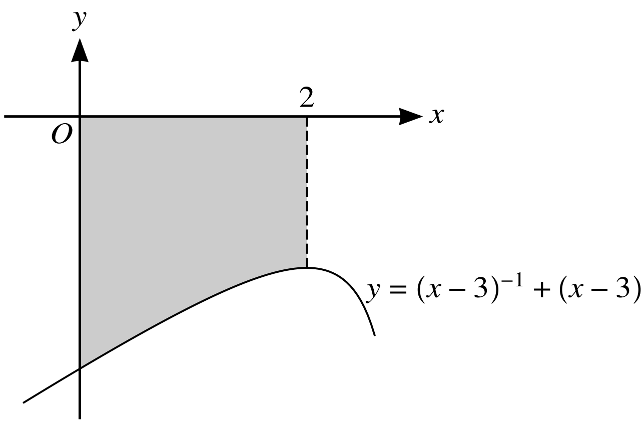
It is now given that $B\,(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB. [5]

11 A curve has equation $y = (kx - 3)^{-1} + (kx - 3)$, where k is a non-zero constant.

(i) Find the x-coordinates of the stationary points in terms of k, and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x-axis, the y-axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x-axis. [5]