

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

## MATHEMATICS

9709/13
Paper 1 Pure Mathematics 1 (P1)

October/November 2013
1 hour 45 minutes

## Additional Materials: Answer Booklet/Paper

 Graph Paper List of Formulae (MF9)
## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 Solve the inequality $x^{2}-x-2>0$.

2 A curve has equation $y=\mathrm{f}(x)$. It is given that $\mathrm{f}^{\prime}(x)=x^{-\frac{3}{2}}+1$ and that $\mathrm{f}(4)=5$. Find $\mathrm{f}(x)$.

3 The point $A$ has coordinates $(3,1)$ and the point $B$ has coordinates $(-21,11)$. The point $C$ is the mid-point of $A B$.
(i) Find the equation of the line through $A$ that is perpendicular to $y=2 x-7$.
(ii) Find the distance $A C$.

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The diagram shows a pyramid $O A B C$ in which the edge $O C$ is vertical. The horizontal base $O A B$ is a triangle, right-angled at $O$, and $D$ is the mid-point of $A B$. The edges $O A, O B$ and $O C$ have lengths of 8 units, 6 units and 10 units respectively. The unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{O A}, \overrightarrow{O B}$ and $\overrightarrow{O C}$ respectively.
(i) Express each of the vectors $\overrightarrow{O D}$ and $\overrightarrow{C D}$ in terms of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$.
(ii) Use a scalar product to find angle $O D C$.

5 (a) In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio.
(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference.


The diagram shows sector $O A B$ with centre $O$ and radius 11 cm . Angle $A O B=\alpha$ radians. Points $C$ and $D$ lie on $O A$ and $O B$ respectively. Arc $C D$ has centre $O$ and radius 5 cm .
(i) The area of the shaded region $A B D C$ is equal to $k$ times the area of the unshaded region $O C D$. Find $k$.
(ii) The perimeter of the shaded region $A B D C$ is equal to twice the perimeter of the unshaded region $O C D$. Find the exact value of $\alpha$.

7 (a) Find the possible values of $x$ for which $\sin ^{-1}\left(x^{2}-1\right)=\frac{1}{3} \pi$, giving your answers correct to 3 decimal places.
(b) Solve the equation $\sin \left(2 \theta+\frac{1}{3} \pi\right)=\frac{1}{2}$ for $0 \leqslant \theta \leqslant \pi$, giving $\theta$ in terms of $\pi$ in your answers.

8 (i) Find the coefficient of $x^{8}$ in the expansion of $\left(x+3 x^{2}\right)^{4}$.
(ii) Find the coefficient of $x^{8}$ in the expansion of $\left(x+3 x^{2}\right)^{5}$.
(iii) Hence find the coefficient of $x^{8}$ in the expansion of $\left[1+\left(x+3 x^{2}\right)\right]^{5}$.

9 A curve has equation $y=\frac{k^{2}}{x+2}+x$, where $k$ is a positive constant. Find, in terms of $k$, the values of $x$ for which the curve has stationary points and determine the nature of each stationary point.

10 The function f is defined by $\mathrm{f}: x \mapsto x^{2}+4 x$ for $x \geqslant c$, where $c$ is a constant. It is given that f is a one-one function.
(i) State the range of f in terms of $c$ and find the smallest possible value of $c$.

The function g is defined by $\mathrm{g}: x \mapsto a x+b$ for $x \geqslant 0$, where $a$ and $b$ are positive constants. It is given that, when $c=0, \operatorname{gf}(1)=11$ and $\mathrm{fg}(1)=21$.
(ii) Write down two equations in $a$ and $b$ and solve them to find the values of $a$ and $b$.


The diagram shows the curve $y=\sqrt{ }\left(x^{4}+4 x+4\right)$.
(i) Find the equation of the tangent to the curve at the point $(0,2)$.
(ii) Show that the $x$-coordinates of the points of intersection of the line $y=x+2$ and the curve are given by the equation $(x+2)^{2}=x^{4}+4 x+4$. Hence find these $x$-coordinates.
(iii) The region shaded in the diagram is rotated through $360^{\circ}$ about the $x$-axis. Find the volume of revolution.

