UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/12   Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2011 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
             B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only – often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	**(i)** $(2-y)^5 = 32 - 80y + 80y^2$	B2,1 [2]	-1 for each error. Accept 2^5.
	(ii) $(2-(2x-x^2))^5$ "y" = "$2x-x^2$"	M1	Allow for $y = 2x + x^2$
	$\to 80 + 320 = 400$	M1	Needs to consider exactly 2 terms.
		A1 [3]	CO – accept $400x^2$, accept full expansion if $400x^2$ is part of it.
2	$f: x \mapsto 3x + a$, $g: x \mapsto b - 2x$		
	(i) $f^2(x) = 3(3x+a) + a$	B1	Must be correct – unsimplified ok
	$f^2(2) = 18 + 4a = 10 \to a = -2$	B1	co
	$g^{-1}(x) = \dfrac{b-x}{2} \to \dfrac{b-2}{2} = 3$ $b = 8$	M1	Correct method leading to a value for b
	or $g(3) = 2 \to b - 6 = 2$ $b = 8$	A1 [4]	co
	(ii) $fg(x) = 3(b-2x) + a$	M1	Must be fg not gf.
	$= 22 - 6x$	A1√ [2]	√ on a and b $(3b + a - 6x)$ must be two term answer.
3	$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k}$	M1	Use of $x_1 x_2 + y_1 y_2 + z_1 z_2$
	(i) $\overrightarrow{OA} \cdot \overrightarrow{OB} = 10 + 7 + 2p$	DM1	....$=0$
	$= 0 \to p = -8\tfrac{1}{2}$	A1 [3]	co
	(ii) $\mathbf{AB} = -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$	B1	co (accept negative)
	Modulus $= \sqrt{(9+36+4)}$	M1	For modulus
	Magnitude 28 $\to$ 28 $\times$unit vector	M1	Scales by $\times 28 \div$ modulus.
	$\to -12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}$.	A1 [4]	Co – could leave as "$4 \times \ldots$".
4	**(i)** $y^2 + 2x = 13$, $2y + x = 8$	M1	Complete elimination of x or y
	$\to y^2 - 4y + 3 = 0$, $x^2 - 8x + 12 = 0$	A1	co (allow multiples) – needs 3 terms
		DM1	Solution of quadratic $= 0$
	$\to (2, 3)$ and $(6, 1)$	A1 [4]	Needs all 4 coordinates.
	(ii) Removes $x \to y^2 + 2(k - 2y) = 13$	M1	Complete elimination of x or y.
	Uses $b^2 - 4ac$ on "quadratic $= 0$)	DM1	Use of discriminant $=0, <0$ or >0
	$\to k = 8\tfrac{1}{2}$	A1 [3]	Co
	or $\dfrac{dy}{dx} = -\tfrac{1}{2} = \dfrac{-1}{y} \to y=2, x=4\tfrac{1}{2}, k= 8\tfrac{1}{2}$		(M1 equating m of line and curve M1 x to y A1 for k)

5 (i)

B1	$y = \sin x$ (0,0). $(\pi,0)$ + curve
B1	$y = \cos 2x$ One full cycle.
B1	$y = \cos 2x$ starts and finishes at (0, 1) and oscillates between −1 and +1.
[3]	Do not penalise graphs from 0 to 360.

(ii) Evidence of sin 30 = cos 60 = 0.5
Other root is 150º

B1	co
B1	co
[2]	

(iii) $0 \le x < 30$ and $150 < x \le 180$
(x < 30 or x > 150 ok)

B1 B1√	Condone < or ≤ throughout
[2]	

6 (i) D to $AX = 6\sin\frac{\pi}{3} = 6\sqrt{3} \div 2$

E to $AX = 10\sin\theta$

Equate these → $\theta = \sin^{-1}\dfrac{3\sqrt{3}}{10}$.

B1	co Needs –√3÷2 not just 3√3.
B1	co
	Correct method. ag.
B1	Use of decimals loses this B mark.
[3]	

(ii) Arc $DX = 6.\frac{1}{3}\pi = 2\pi$
Arc $EX = 10 \times 0.5464 = 5.464$
Horizontal steps = $6\cos\frac{1}{3}\pi$ and $10\cos\theta$
$DE = 10 + 6 - 6\cos\frac{1}{3}\pi - 10\cos\theta$
Perimeter = arc DX + arc BX + DE
→ 16.20

B1	co
M1	Use of $s = r\theta$ radians.
M1	Attempt at both steps needed
M1	Full method for DE.
A1	Co – must be exactly 16.20, not more or
[5]	less places.

7 $\dfrac{dy}{dx} = 5 - \dfrac{8}{x^2}$, Normal $3y + x = 17$

(i) Gradient of line = $-\frac{1}{3}$

$\dfrac{dy}{dx} = 3$ → $x = 2$, $y = 5$

B1	co
M1	Use of $m_1 m_2 = -1$
DM1	DM1 solution. A1 co.
A1	
[4]	

(ii) $y = 5x + 8x^{-1}(+c)$
Uses (2, 5) → $c = -9$

B1 B1	co.co. doesn't need +c.
M1 A1	Use of +c following integration. co.
[4]	

© University of Cambridge International Examinations 2011

8 $y = \sqrt{8x - x^2}$

(i) $\dfrac{dy}{dx} = \frac{1}{2}(8x - x^2)^{-\frac{1}{2}} \times (8 - 2x)$ B1   B1 for everything but $\times(8\text{-}2x)$

                          B1   B1 for $\times (8{-}2x)$, even if B0

     $= 0$ when $x = 4$.   M1   Sets to 0 + attempt at solution.

     $\rightarrow$ (4, 4)   A1   Co – A0 if fortuitous because of B0

                            [4]   earlier.

(ii) $y = 0$ when $x = 0$ or 8   B1

     Vol $= \pi \displaystyle\int (8x - x^2)\,dx$ Anywhere

     $= \pi \left[4x^2 - \dfrac{x^3}{3} \right]$ B2,1 -1 for each error (not including π)

     $\rightarrow \quad \dfrac{256\pi}{3}$ B1

                            [4]   co

---

9 (i) Gradient of $AC = \frac{1}{2}$ B1   co

     Gradient of $BD = -2$ M1   Use of $m_1 m_2 = -1$ with AC

     Eqn of BD is $y - 6 = -2(x - 3)$ M1   Correct formula for straight line

     Eqn of AC is $y + 1 = \frac{1}{2}(x + 1)$ M1   Solution.

     Sim eqns $\rightarrow M(5, 2)$ A1   co

     Vector move – or midpoint back

     $\rightarrow D(7, -2)$ M1 A1√   Correct method. √ on M.

                            [7]

(ii) Ratio of $AM : MC = \sqrt{45} : \sqrt{20}$ M1   Correct distance formula.

     or Vector step $\rightarrow$ 3 : 2   A1   Looks at the two x or y steps.

                            [2]   Must be numerical, 1.5 ok, not as roots

---

10 (a) $a = -15$, $n = 25$

     (i) Use of $S_n \rightarrow d = 3$.   M1 A1   Must be correct formula. co

                            [2]

     (ii) Last term $= a + 24d$ M1   Must be $a + 24d$

          $\rightarrow$ 57   A1√   √ for his d.

         (or $525 = \frac{1}{2} \times 25 \times (-15 + l) \rightarrow l = 57$)   [2]

     (iii) Positive terms are 3, 6, ...57

          Either $a = 0$ or 3, $n = 19$ or 20   M1   Correct use of formula for S_n.

          Use of S_{19} or S_{20}

          $\rightarrow$ 570   A1   co

                           [2]

   (b) $r = 1.05$ B1   In either part **(i)** or **(ii)**.

     (i) 11th term $= ar^{10} = \$6516$ or $\$6520$ B1   co

                            [2]

     (ii) $S_{11} = \dfrac{4000 \times (1.05^{11} - 1)}{.05}$ M1   Correct sum formula with their r.

         $= \$56800$ or (56827) A1   co

                            [2]