UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1454869105

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

UNIVERSITY of CAMBRIDGE
International Examinations

1 Find $\int \left(x + \frac{1}{x}\right)^2 \, dx$. [3]

2 In the expansion of $(1 + ax)^6$, where a is a constant, the coefficient of x is -30. Find the coefficient of x^3. [4]

3 Functions f and g are defined for $x \in \mathbb{R}$ by

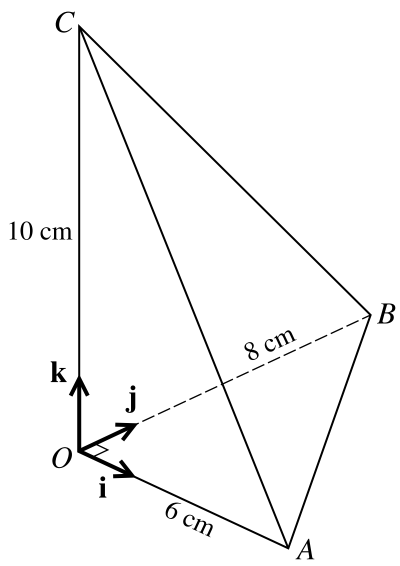
$$\mathrm{f} : x \mapsto 2x + 3,$$
$$\mathrm{g} : x \mapsto x^2 - 2x.$$

Express $\mathrm{gf}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [5]

4 **(i)** Prove the identity $\dfrac{\sin x \tan x}{1 - \cos x} \equiv 1 + \dfrac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\dfrac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0° \leqslant x \leqslant 360°$. [3]

5



The diagram shows a pyramid $OABC$ with a horizontal base OAB where $OA = 6$ cm, $OB = 8$ cm and angle $AOB = 90°$. The point C is vertically above O and $OC = 10$ cm. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC as shown.

Use a scalar product to find angle ACB. [6]

6 **(a)** The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

7 A function f is defined by $\mathrm{f} : x \mapsto 3 - 2\tan\left(\frac{1}{2}x\right)$ for $0 \leqslant x < \pi$.

(i) State the range of f. [1]

(ii) State the exact value of $\mathrm{f}\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression, in terms of x, for $\mathrm{f}^{-1}(x)$. [3]

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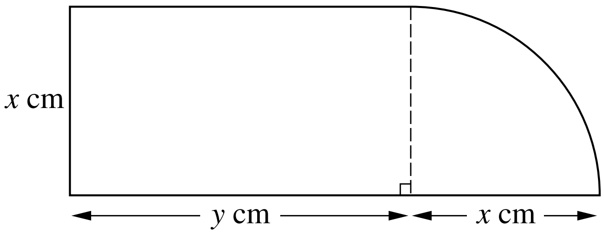


The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

(i) Express y in terms of x. [2]

(ii) Show that the area of the plate, A cm^2, is given by $A = 30x - x^2$. [2]

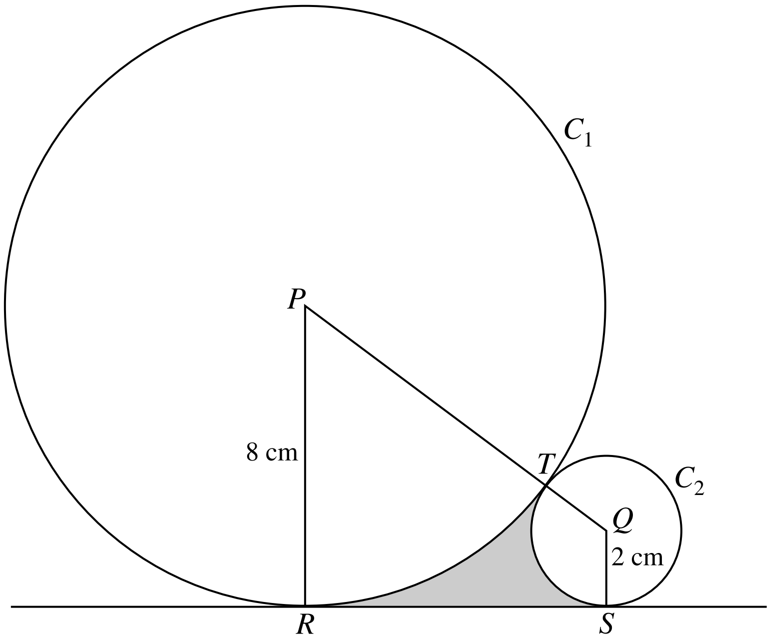
Given that x can vary,

(iii) find the value of x at which A is stationary, [2]

(iv) find this stationary value of A, and determine whether it is a maximum or a minimum value. [2]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows two circles, C_1 and C_2, touching at the point T. Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

10 The equation of a curve is $y = 3 + 4x - x^2$.

(i) Show that the equation of the normal to the curve at the point (3, 6) is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B, find the coordinates of the mid-point of AB. [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

11 The equation of a curve is $y = \dfrac{9}{2 - x}$.

(i) Find an expression for $\dfrac{dy}{dx}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through $360°$ about the x-axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]