



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education  
Advanced Subsidiary Level and Advanced Level



**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**May/June 2010**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
The use of an electronic calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 75.  
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.

- 1 (i) Show that the equation

$$3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$$

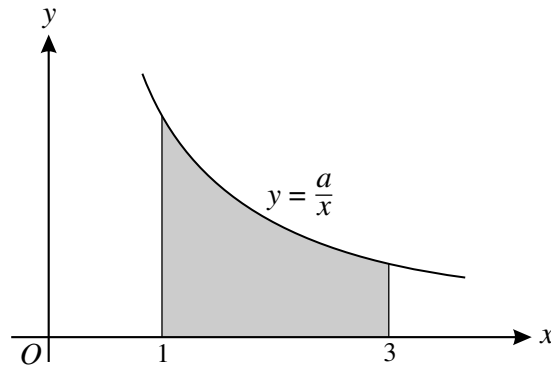
can be written in the form  $\tan x = -\frac{3}{4}$ .

[2]

- (ii) Solve the equation  $3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$ , for  $0^\circ \leq x \leq 360^\circ$ .

[2]

2



The diagram shows part of the curve  $y = \frac{a}{x}$ , where  $a$  is a positive constant. Given that the volume obtained when the shaded region is rotated through  $360^\circ$  about the  $x$ -axis is  $24\pi$ , find the value of  $a$ .

[4]

- 3 The functions  $f$  and  $g$  are defined for  $x \in \mathbb{R}$  by

$$f : x \mapsto 4x - 2x^2,$$

$$g : x \mapsto 5x + 3.$$

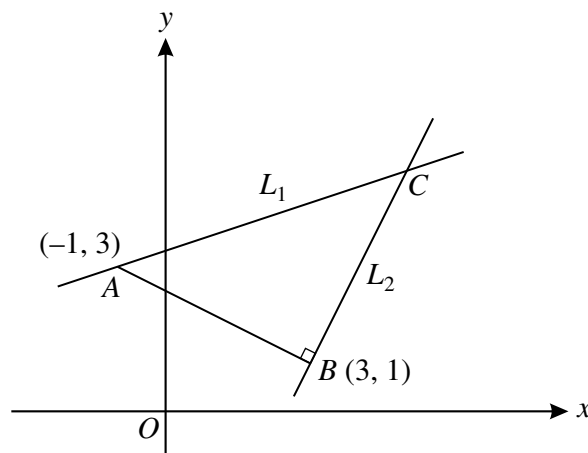
- (i) Find the range of  $f$ .

[2]

- (ii) Find the value of the constant  $k$  for which the equation  $gf(x) = k$  has equal roots.

[3]

4



In the diagram,  $A$  is the point  $(-1, 3)$  and  $B$  is the point  $(3, 1)$ . The line  $L_1$  passes through  $A$  and is parallel to  $OB$ . The line  $L_2$  passes through  $B$  and is perpendicular to  $AB$ . The lines  $L_1$  and  $L_2$  meet at  $C$ . Find the coordinates of  $C$ .

[6]

5 Relative to an origin  $O$ , the position vectors of the points  $A$  and  $B$  are given by

$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ p \end{pmatrix}.$$

(i) Find the value of  $p$  for which  $\overrightarrow{OA}$  is perpendicular to  $\overrightarrow{OB}$ . [2]

(ii) Find the values of  $p$  for which the magnitude of  $\overrightarrow{AB}$  is 7. [4]

6 (i) Find the first 3 terms in the expansion of  $(1 + ax)^5$  in ascending powers of  $x$ . [2]

(ii) Given that there is no term in  $x$  in the expansion of  $(1 - 2x)(1 + ax)^5$ , find the value of the constant  $a$ . [2]

(iii) For this value of  $a$ , find the coefficient of  $x^2$  in the expansion of  $(1 - 2x)(1 + ax)^5$ . [3]

7 (a) Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]

(b) A geometric progression has a common ratio of  $-\frac{2}{3}$  and the sum of the first 3 terms is 35. Find

(i) the first term of the progression, [3]

(ii) the sum to infinity. [2]

8 A solid rectangular block has a square base of side  $x$  cm. The height of the block is  $h$  cm and the total surface area of the block is  $96 \text{ cm}^2$ .

(i) Express  $h$  in terms of  $x$  and show that the volume,  $V \text{ cm}^3$ , of the block is given by

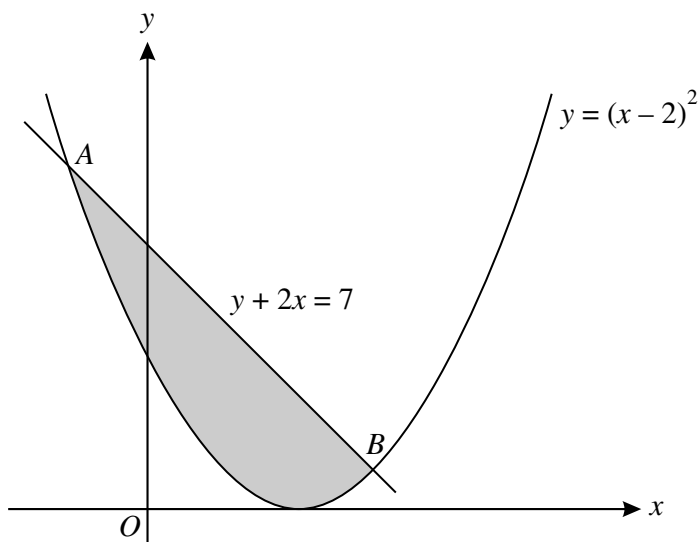
$$V = 24x - \frac{1}{2}x^3. \quad [3]$$

Given that  $x$  can vary,

(ii) find the stationary value of  $V$ , [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]

[Questions 9, 10 and 11 are printed on the next page.]



The diagram shows the curve  $y = (x - 2)^2$  and the line  $y + 2x = 7$ , which intersect at points  $A$  and  $B$ . Find the area of the shaded region. [8]

10 The equation of a curve is  $y = \frac{1}{6}(2x - 3)^3 - 4x$ .

(i) Find  $\frac{dy}{dx}$ . [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the  $y$ -axis. [3]

(iii) Find the set of values of  $x$  for which  $\frac{1}{6}(2x - 3)^3 - 4x$  is an increasing function of  $x$ . [3]

11 The function  $f : x \mapsto 4 - 3 \sin x$  is defined for the domain  $0 \leq x \leq 2\pi$ .

(i) Solve the equation  $f(x) = 2$ . [3]

(ii) Sketch the graph of  $y = f(x)$ . [2]

(iii) Find the set of values of  $k$  for which the equation  $f(x) = k$  has no solution. [2]

The function  $g : x \mapsto 4 - 3 \sin x$  is defined for the domain  $\frac{1}{2}\pi \leq x \leq A$ .

(iv) State the largest value of  $A$  for which  $g$  has an inverse. [1]

(v) For this value of  $A$ , find the value of  $g^{-1}(3)$ . [2]

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