



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education  
Advanced Subsidiary Level and Advanced Level



**MATHEMATICS**

**9709/01**

Paper 1 Pure Mathematics 1 (P1)

**May/June 2009**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper  
Graph Paper  
List of Formulae (MF9)



**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

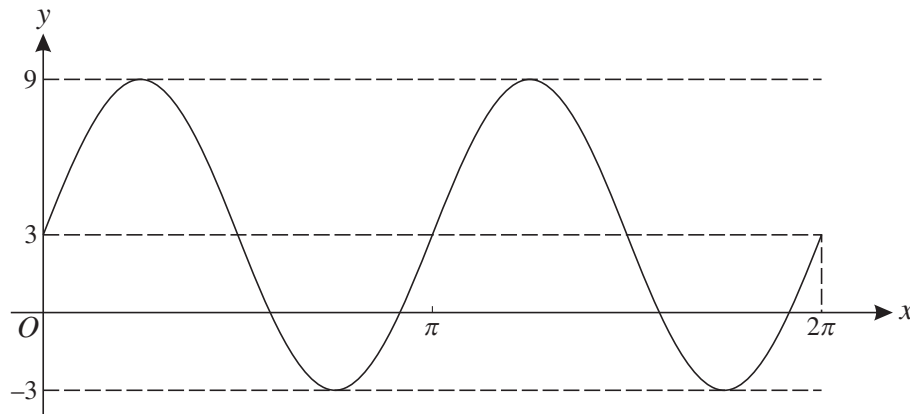
Answer **all** the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
The use of an electronic calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.  
The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 75.  
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.

- 1 Prove the identity  $\frac{\sin x}{1 - \sin x} - \frac{\sin x}{1 + \sin x} \equiv 2 \tan^2 x$ . [3]
- 2 Find the set of values of  $k$  for which the line  $y = kx - 4$  intersects the curve  $y = x^2 - 2x$  at two distinct points. [4]
- 3 (i) Find the first 3 terms in the expansion of  $(2 + 3x)^5$  in ascending powers of  $x$ . [3]
- (ii) Hence find the value of the constant  $a$  for which there is no term in  $x^2$  in the expansion of  $(1 + ax)(2 + 3x)^5$ . [2]

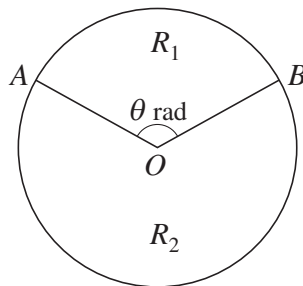
4



The diagram shows the graph of  $y = a \sin(bx) + c$  for  $0 \leq x \leq 2\pi$ .

- (i) Find the values of  $a$ ,  $b$  and  $c$ . [3]
- (ii) Find the smallest value of  $x$  in the interval  $0 \leq x \leq 2\pi$  for which  $y = 0$ . [3]

5



The diagram shows a circle with centre  $O$ . The circle is divided into two regions,  $R_1$  and  $R_2$ , by the radii  $OA$  and  $OB$ , where angle  $AOB = \theta$  radians. The perimeter of the region  $R_1$  is equal to the length of the major arc  $AB$ .

- (i) Show that  $\theta = \pi - 1$ . [3]
- (ii) Given that the area of region  $R_1$  is  $30 \text{ cm}^2$ , find the area of region  $R_2$ , correct to 3 significant figures. [4]

- 6 Relative to an origin  $O$ , the position vectors of the points  $A$  and  $B$  are given by

$$\vec{OA} = 2\mathbf{i} - 8\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \vec{OB} = 7\mathbf{i} + 2\mathbf{j} - \mathbf{k}.$$

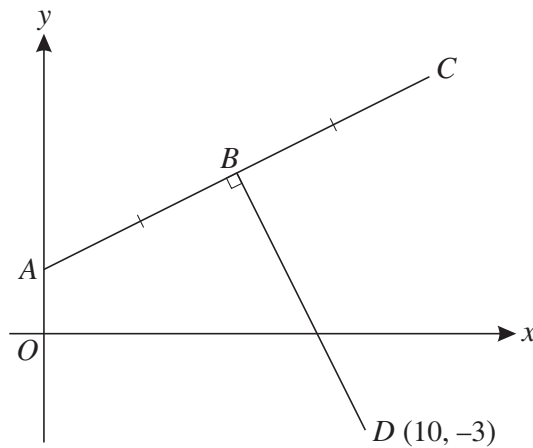
(i) Find the value of  $\vec{OA} \cdot \vec{OB}$  and hence state whether angle  $AOB$  is acute, obtuse or a right angle. [3]

(ii) The point  $X$  is such that  $\vec{AX} = \frac{2}{5}\vec{AB}$ . Find the unit vector in the direction of  $OX$ . [4]

- 7 (a) Find the sum to infinity of the geometric progression with first three terms  $0.5$ ,  $0.5^3$  and  $0.5^5$ . [3]

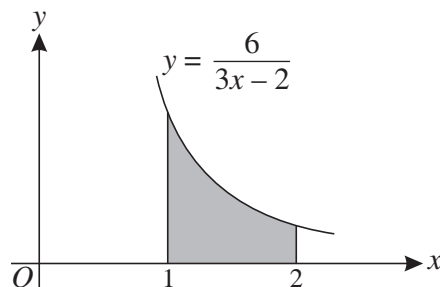
(b) The first two terms in an arithmetic progression are  $5$  and  $9$ . The last term in the progression is the only term which is greater than  $200$ . Find the sum of all the terms in the progression. [4]

8



The diagram shows points  $A$ ,  $B$  and  $C$  lying on the line  $2y = x + 4$ . The point  $A$  lies on the  $y$ -axis and  $AB = BC$ . The line from  $D(10, -3)$  to  $B$  is perpendicular to  $AC$ . Calculate the coordinates of  $B$  and  $C$ . [7]

9



The diagram shows part of the curve  $y = \frac{6}{3x - 2}$ .

(i) Find the gradient of the curve at the point where  $x = 2$ . [3]

(ii) Find the volume obtained when the shaded region is rotated through  $360^\circ$  about the  $x$ -axis, giving your answer in terms of  $\pi$ . [5]

10 The function  $f$  is defined by  $f : x \mapsto 2x^2 - 12x + 13$  for  $0 \leq x \leq A$ , where  $A$  is a constant.

(i) Express  $f(x)$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are constants. [3]

(ii) State the value of  $A$  for which the graph of  $y = f(x)$  has a line of symmetry. [1]

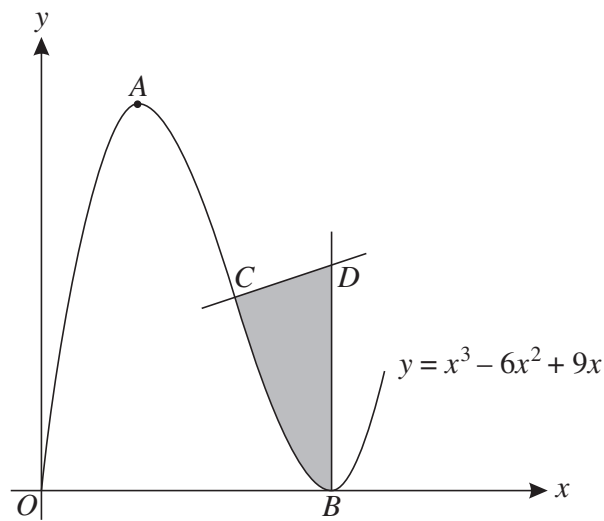
(iii) When  $A$  has this value, find the range of  $f$ . [2]

The function  $g$  is defined by  $g : x \mapsto 2x^2 - 12x + 13$  for  $x \geq 4$ .

(iv) Explain why  $g$  has an inverse. [1]

(v) Obtain an expression, in terms of  $x$ , for  $g^{-1}(x)$ . [3]

11



The diagram shows the curve  $y = x^3 - 6x^2 + 9x$  for  $x \geq 0$ . The curve has a maximum point at  $A$  and a minimum point on the  $x$ -axis at  $B$ . The normal to the curve at  $C(2, 2)$  meets the normal to the curve at  $B$  at the point  $D$ .

(i) Find the coordinates of  $A$  and  $B$ . [3]

(ii) Find the equation of the normal to the curve at  $C$ . [3]

(iii) Find the area of the shaded region. [5]