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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

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FURTHER MATHEMATICS **9231/21**

Paper 2 **October/November 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **22** printed pages and **2** blank pages.

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1 A particle P oscillates in simple harmonic motion between the points A and B, where $AB = 6$ m. The period of the motion is $\frac{1}{2}\pi$ s. Find the speed of P when it is 2 m from B. [3]

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2 Two uniform small smooth spheres A and B have equal radii and masses $5m$ and $2m$ respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides directly with sphere B which is moving towards it with speed $2u$. The coefficient of restitution between the spheres is e.

(i) Show that the speed of B after the collision is $\frac{1}{7}u(1 + 15e)$ and find an expression for the speed of A. [4]

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In the collision, the speed of A is halved and its direction of motion is reversed.

(ii) Find the value of e. [2]

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(iii) For this collision, find the ratio of the loss of kinetic energy of A to the loss of kinetic energy of B. [3]

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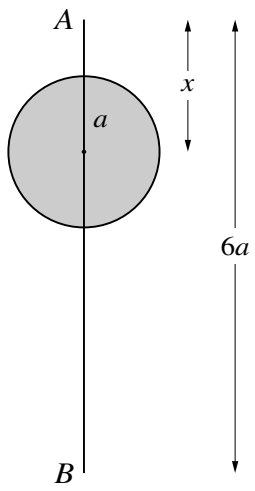

A uniform disc, of radius a and mass $2M$, is attached to a thin uniform rod AB of length $6a$ and mass M. The rod lies along a diameter of the disc, so that the centre of the disc is a distance x from A (see diagram).

(i) Find the moment of inertia of the object, consisting of disc and rod, about a fixed horizontal axis l through A and perpendicular to the plane of the disc. [4]

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The object is free to rotate about the axis l. The object is held with AB horizontal and is released from rest. When AB makes an angle θ with the vertical, where $\cos\theta = \frac{3}{5}$, the angular speed of the object is $\sqrt{\left(\frac{2g}{5a}\right)}$.

(ii) Find the possible values of x. [5]

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4 A uniform rod AB of length $4a$ and weight W is smoothly hinged to a vertical wall at the end A. The rod is held at an angle θ above the horizontal by a light elastic string. One end of the string is attached to the point C on the rod, where $AC = 3a$. The other end of the string is attached to a point D on the wall, with D vertically above A and such that angle $ACD = 2\theta$. A particle of weight $\frac{1}{2}W$ is attached to the rod at B. It is given that $\tan\theta = \frac{8}{15}$.

(i) Show that the tension in the string is $\frac{17}{12}W$. [4]

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(ii) Find the magnitude and direction of the reaction at the hinge. [5]

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(iii) Given that the natural length of the string is $2a$, find its modulus of elasticity. [2]

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5 The fixed points A and B are on a smooth horizontal surface with $AB = 2.6$ m. One end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N, is attached to A. The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B; its other end is attached to P. The system is in equilibrium with P on the surface at the point E.

(i) Show that $AE = 1.4$ m. [4]

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The particle P is now displaced slightly from E, along the line AB.

(ii) Show that, in the subsequent motion, P performs simple harmonic motion. [5]

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(iii) Given that the period of the motion is $\frac{1}{7}\pi$ s, find the value of λ. [3]

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6 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{80}\left(3\sqrt{x} - \frac{8}{\sqrt{x}}\right) & 4 \leqslant x \leqslant 16, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

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The random variable Y is defined by $Y = \sqrt{X}$.

(ii) Find the probability density function of Y. [3]

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7 The random variable T is the lifetime, in hours, of a particular type of battery. It is given that T has a negative exponential distribution with mean 500 hours.

(i) Write down the probability density function of T. [1]

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(ii) Find the probability that a randomly chosen battery of this type has a lifetime of more than 750 hours. [3]

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(iii) Find the median value of T. [3]

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8 The weekly salaries of employees at two large electronics companies, A and B, are being compared. The weekly salaries of an employee from company A and an employee from company B are denoted by $\$x$ and $\$y$ respectively. A random sample of 50 employees from company A and a random sample of 40 employees from company B give the following summarised data.

$$\Sigma x = 5120 \qquad \Sigma x^2 = 531\,000 \qquad \Sigma y = 3760 \qquad \Sigma y^2 = 375\,135$$

(i) The population mean salaries of employees from companies A and B are denoted by $\$\mu_A$ and $\$\mu_B$ respectively. Using a 5% significance level, test the null hypothesis $\mu_A = \mu_B$ against the alternative hypothesis $\mu_A \neq \mu_B$. [8]

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(ii) State, with a reason, whether any assumptions about the distributions of employees' salaries are needed for the test in part **(i)**. [1]

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9 There are a large number of students at a particular college. The heights, in metres, of a random sample of 8 students are as follows.

1.75 1.72 1.62 1.70 1.82 1.75 1.68 1.84

You may assume that heights of students are normally distributed.

(i) Test, at the 5% significance level, whether the population mean height of students at this college is greater than 1.70 metres. [7]

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(ii) Find a 95% confidence interval for the population mean height of students at this college. [3]

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10 For a random sample of 10 observations of pairs of values (x, y), the equation of the regression line of y on x is $y = 1.1664 + 0.4604x$. It is given that

$$\Sigma x^2 = 1419.98 \qquad \text{and} \qquad \Sigma y^2 = 439.68.$$

The mean value of y is 6.24.

(i) Find the equation of the regression line of x on y. [6]

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(ii) Find the product moment correlation coefficient. [2]

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(iii) Test at the 5% significance level whether there is evidence of positive correlation between the two variables. [4]

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11 Answer only **one** of the following two alternatives.

EITHER

A particle P of mass m is free to move on the smooth inner surface of a fixed hollow sphere of radius a. The centre of the sphere is O and the point C is on the inner surface of the sphere, vertically below O. The points A and B on the inner surface of the sphere are the ends of a diameter of the sphere. The diameter AOB makes an acute angle α with the vertical, where $\cos\alpha = \frac{4}{5}$, with A below the horizontal level of B. The particle is projected from A with speed u, and moves along the inner surface of the sphere towards C. The normal reaction forces on the particle at A and C are in the ratio 8 : 9.

(i) Show that $u^2 = 4ag$. [6]

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(ii) Determine whether P reaches B without losing contact with the inner surface of the sphere. [6]

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OR

A machine is used to produce metal rods. When the machine is working efficiently, the lengths, x cm, of the rods have a normal distribution with mean 150 cm and standard deviation 1.2 cm. The machine is checked regularly by taking random samples of 200 rods. The latest results are shown in the following table.

Interval	$146 \leqslant x < 147$	$147 \leqslant x < 148$	$148 \leqslant x < 149$	$149 \leqslant x < 150$	$150 \leqslant x < 151$	$151 \leqslant x < 152$	$152 \leqslant x < 153$	$153 \leqslant x < 154$
Observed frequency	1	2	23	52	69	36	15	2

As a first check, the sample is used to calculate an estimate for the mean.

(i) Show that an estimate for the mean from this sample is close to 150 cm. [2]

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As a second check, the results are tested for goodness of fit of the normal distribution with mean 150 cm and standard deviation 1.2 cm. The relevant expected frequencies, found using the normal distribution function given in the List of Formulae (MF10), are shown in the following table.

Interval	$x < 147$	$147 \leqslant x < 148$	$148 \leqslant x < 149$	$149 \leqslant x < 150$	$150 \leqslant x < 151$	$151 \leqslant x < 152$	$152 \leqslant x < 153$	$153 \leqslant x$
Observed frequency	1	2	23	52	69	36	15	2
Expected frequency	1.24	8.32	30.94	59.50	59.50	30.94	8.32	1.24

(ii) Show how the expected frequency for $151 \leqslant x < 152$ is obtained. [3]

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(iii) Test, at the 5% significance level, the goodness of fit of the normal distribution to the results. [7]

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Additional Page

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