Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

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FURTHER MATHEMATICS **9231/22**

Paper 2 **October/November 2016**

3 hours

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages, **2** blank pages and **1** insert.

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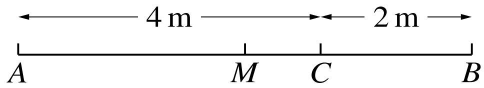
1



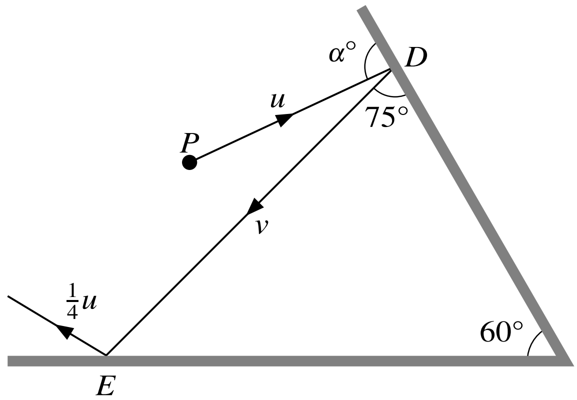
The point C is on the fixed line l. Points A and B on l are such that $AC = 4$ m and $CB = 2$ m, with C between A and B. The point M is the mid-point of AB (see diagram). A particle P of mass m oscillates between A and B in simple harmonic motion. When P is at C, its speed is 4 m s^{-1}. Find

(i) the magnitude of the maximum acceleration of P, [3]

(ii) the number of complete oscillations made by P in one minute, [2]

(iii) the time that P takes to travel directly from A to C. [3]

2

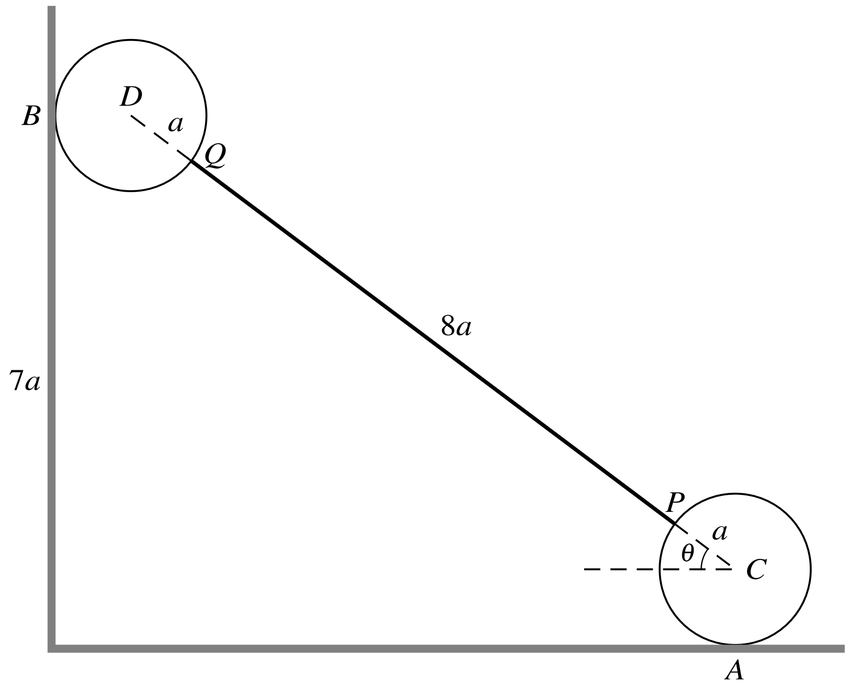


Two smooth vertical walls each with their base on a smooth horizontal surface intersect at an angle of 60°. A small smooth sphere P is moving on the horizontal surface with speed u when it collides with the first vertical wall at the point D. The angle between the direction of motion of P and the wall is $\alpha°$ before the collision and 75° after the collision. The speed of P after this collision is v and the coefficient of restitution between P and the first wall is e. Sphere P then collides with the second vertical wall at the point E. The speed of P after this second collision is $\frac{1}{4}u$ (see diagram). The coefficient of restitution between P and the second wall is $\frac{3}{4}$.

(i) By considering the collision at E, show that $v = \frac{\sqrt{2}}{5}u$. [5]

(ii) Find the value of α and the value of e. [5]

3



The end P of a uniform rod PQ, of weight kW and length $8a$, is rigidly attached to a point on the surface of a uniform sphere with centre C, weight W and radius a. The end Q is rigidly attached to a point on the surface of an identical sphere with centre D. The points C, P, Q and D are in a straight line. The object consisting of the rod and two spheres rests with one sphere in contact with a rough horizontal surface, at the point A, and the other sphere in contact with a smooth vertical wall, at the point B. The angle between CD and the horizontal is θ. The point B is at a height of $7a$ above the base of the wall (see diagram). The points A, B, C, D, P and Q are all in the same vertical plane.

(i) Show that $\sin\theta = \frac{3}{5}$. [1]

The object is in limiting equilibrium and the coefficient of friction at A is μ.

(ii) Find the numerical value of μ. [7]

(iii) Given that the resultant force on the object at A is $W\sqrt{(65)}$, show that $k = 5$. [3]

4 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is held vertically above O with the string taut and then projected horizontally with speed $\sqrt{\left(\frac{13}{3}ag\right)}$. It begins to move in a vertical circle with centre O. When P is at its lowest point, it collides with a stationary particle of mass λm. The two particles coalesce.

(i) Show that the speed of the combined particle immediately after the impact is $\dfrac{5}{\lambda + 1}\sqrt{\left(\frac{1}{3}ag\right)}$. [4]

In the subsequent motion, the string becomes slack when the combined particle is at a height of $\frac{1}{3}a$ above the level of O.

(ii) Find the value of λ. [6]

(iii) Find, in terms of m and g, the instantaneous change in the tension in the string as a result of the collision. [4]

5 The distance, X km, completed by a new car before any mechanical fault occurs has distribution function F given by

$$F(x) = \begin{cases} 1 - e^{-ax} & x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a positive constant. The mean value of X is 10 000. Find

(i) the value of a, [1]

(ii) the probability that a new car completes less than 15 000 km before any mechanical fault occurs. [2]

The probability that a new car completes at least d km before any mechanical fault occurs is 0.75.

(iii) Find the value of d. [3]

6 A random sample of 8 observations of a normal random variable X has mean $\bar{x}$, where

$$\bar{x} = 6.246 \quad \text{and} \quad \Sigma(x - \bar{x})^2 = 0.784.$$

Test, at the 5% significance level, whether the population mean of X is less than 6.44. [7]

7 The random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{6}x & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the distribution function of X. [3]

The random variable Y is defined by $Y = X^3$. Find

(ii) the probability density function of Y, [2]

(iii) the value of k for which $P(Y \geqslant k) = \frac{7}{12}$. [3]

8 The amounts spent on the weekly food shopping by families in the big city P and the small town Q are to be compared. The amounts spent, in dollars, in P and Q are denoted by x and y respectively. For a random sample of 60 families in P and a random sample of 50 families in Q, the amounts are summarised as follows.

$$\Sigma x = 9600 \qquad \Sigma x^2 = 1\,560\,000 \qquad \Sigma y = 7200 \qquad \Sigma y^2 = 1\,052\,500$$

Assuming a common population variance, find

(i) a pooled estimate for the population variance, [4]

(ii) a 95% confidence interval for the difference in the population means in P and Q. [5]

9 The number of visitors arriving at an art exhibition is recorded for each 10-minute period of time during the ten hours that it is open on a particular day. The results are as follows.

Number of visitors in a 10-minute period	0	1	2	3	4	5	6	7	8	$\geqslant 9$
Number of 10-minute periods	2	2	12	8	11	13	4	7	1	0

(i) Calculate the mean and variance for this sample and explain whether your answers support a suggestion that a Poisson distribution might be a suitable model for the number of visitors in a 10-minute period. [3]

(ii) Use an appropriate Poisson distribution to find the two expected frequencies missing from the following table. [2]

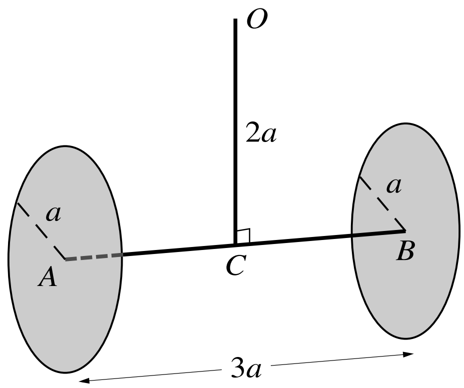
Number of visitors in a 10-minute period	0	1	2	3	4	5	6	7	8	$\geqslant 9$
Expected number of 10-minute periods	1.10		8.79		11.72	9.38	6.25	3.57	1.79	1.28

(iii) Test, at the 10% significance level, the goodness of fit of this Poisson distribution to the data. [8]

[Question 10 is printed on the next page.]

10 Answer only **one** of the following two alternatives.

EITHER



A thin uniform rod AB has mass $2m$ and length $3a$. Two identical uniform discs each have mass $\frac{1}{2}m$ and radius a. The centre of one of the discs is rigidly attached to the end A of the rod and the centre of the other disc is rigidly attached to the end B of the rod. The plane of each disc is perpendicular to the rod AB. A second thin uniform rod OC has mass m and length $2a$. The end C of this rod is rigidly attached to the mid-point of AB, with OC perpendicular to AB (see diagram). The object consisting of the two discs and two rods is free to rotate about a horizontal axis l, through O, which is perpendicular to both rods.

(i) Show that the moment of inertia of one of the discs about l is $\frac{13}{4}ma^2$. [3]

(ii) Show that the moment of inertia of the object about l is $\frac{52}{3}ma^2$. [4]

When the object is suspended from O and is hanging in equilibrium, the point C is given a speed of $\sqrt{(2ag)}$ in the direction parallel to AB. In the subsequent motion, the angle through which OC has turned before the object comes to instantaneous rest is θ.

(iii) Show that $\cos\theta = \frac{8}{21}$. [7]

OR

For a random sample, A, of 5 pairs of values of x and y, the equations of the regression lines of y on x and x on y are respectively $y = 4.5 + 0.3x$ and $x = 3y - 13$. Four of the five pairs of data are given in the following table.

x	1	5	7	9
y	5	6	7	7

Find

(i) the fifth pair of values of x and y, [5]

(ii) the value of the product moment correlation coefficient. [2]

A second random sample, B, of 5 pairs of values of x and y is summarised as follows.

$$\Sigma x = 20 \qquad \Sigma x^2 = 100 \qquad \Sigma y = 17 \qquad \Sigma y^2 = 69 \qquad \Sigma xy = 75$$

The two samples, A and B, are combined to form a single random sample of size 10.

(iii) Use this combined sample to test, at the 5% significance level, whether the population product moment correlation coefficient is different from zero. [7]

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