CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9231 FURTHER MATHEMATICS

9231/21 Paper 2, maximum raw mark 100

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through √’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question Number	Mark Scheme Details			Part Mark	Total
1	Find MI of rod *AB* or *CD* about *AD*: Find MI of rod *BC* about *AD*: Find MI of disc about *AD* by perpendicular axes: Find MI of system about *A*:	$I_{AB} = ⅓ Ma^2 + Ma^2$ [$= 4Ma^2/3$] $I_{BC} = M(2a)^2$ [$= 4Ma^2$] $I_{Disc} = ½\ ½\ ⅔\ Ma^2 = Ma^2/6$ $I = 2\ I_{AB} + I_{BC} + I_{Disc}$ $= (41/6)\ Ma^2$	B1 B1 M1 A1 M1 A1	6	[6]
2	Use $T = 2\pi/\omega$ to find ω: Use $v^2 = \omega^2 (A^2 – x^2)$ to find A^2: Use $v^2 = \omega^2 (A^2 – x^2)$ to find v at $x = 6$: [√ on ω] Find time at *A* *or* at *B*, e.g.: Combine correctly to find time from *A* to *B*: *or* Evaluate to 2 d.p.: [√ on ω]	$\omega = 2$ $12^2 = \omega^2 (A^2 – 3^2)$, $A^2 = 45$ $v_B^2 = 2^2 (45 – 6^2)$, $v_B = 6$ [m s^{-1}] ½ sin^{-1} (3/√45) *or* ½ cos^{-1} (3/√45) ½ sin^{-1} (6/√45) *or* ½ cos^{-1} (6/√45) ½ sin^{-1} (6/√45) – ½ sin^{-1} (3/√45) ½ cos^{-1} (3/√45) – ½ cos^{-1} (6/√45) = 0·554 – 0·232 = 0·32 [s]	B1 M1 A1 B1√ M1 A1 M1 A1√	4 4	[8]
3	*EITHER:* Use $\omega^2 = 2\ (d\omega/dt)\ \theta$ to find $d\omega/dt$ [*or a*]: Find eqn of motion for disc: Substitute to find *T*: Find eqn of motion for block: Substitute to find *R*: (g = 9·81 gives R = 32·99) **S.R.**: B1 only for $(4g – R) \times 0{\cdot}2 = 0{\cdot}2^2\ d\omega/dt$ *OR:* Use energy to find *R* (without $d\omega/dt$): (or in terms of $v = 0{\cdot}2\ \omega = 1$) Simplify: Find 2 eqns of motion (or energy for block) Eliminate $d\omega/dt$ to find *T*:	 $5^2 = 2 \times d\omega/dt \times 2$ $d\omega/dt$ = 25/4 *or* 6·25 [a = 1·25] $T \times 0{\cdot}2 = (½ \times 2 \times 0{\cdot}2^2)\ d\omega/dt$ $T = 0{\cdot}2\ d\omega/dt$ = 5/4 *or* 1·25 $4g – R – T = 4 \times 0{\cdot}2\ d\omega/dt$ *or* $4a$ $R = 4g – T – 0{\cdot}8\ d\omega/dt$ = 135/4 *or* 33·7[5] $½\ 0{\cdot}2^2\ \omega^2 + ½\ 4\ (0{\cdot}2\ \omega)^2$ $= (4g – R) \times 0{\cdot}2\ \theta$ $½ + 2 = (4g – R) \times 0{\cdot}4$ $R = 4g – 5/4 – 5$ = 135/4 *or* 33·7[5] $4g – R – T = 4T$, $T = 1{\cdot}25$	 M1 A1 A1 M1 A1 A1 M1 A1 A1 (M1 A1) (A1) (2 × M1 A1) (M1 A1)	9	[9]
4 (i) (ii)	Take moments about *O* (*a* may be cancelled out): Find 2 indep. moment or resln. eqns, e.g.: (M1 needs 2 eqns) Relate R_A, F_A, R_B and F_B : Substitute in above eqns to give e.g.: Solve for *P*: Find R_A / R_B by any valid method, e.g.:	$F_A + F_B = P$ **A.G.** $R_A + F_B = W – P \sin\theta$ $R_B – F_A = P \cos\theta$ $R_A = 2F_A$ *and* $R_B = 2F_B$ $2F_A + F_B = W – 4P/5$ $2F_B – F_A = 3P/5$ [$F_A = 7P/15 = 7W/34$ $F_B = 8P/15 = 8W/34$] $P = 15W/34$ **A.G.** $R_A / R_B = F_A / F_B = 7/8$	M1 A1 M1 B1 A1 A1 M1 A1 M1 A1	2 6 2	[10]

Question Number	Mark Scheme Details			Part Mark	To
5	Use conservation of momentum, e.g.:	$2mv_A + mv_B = 2mu$	B1		
	Use restitution (must be consistent with eqn.): $v_A - v_B = -\frac{2}{3}u$		B1		
	Solve for v_A and v_B (A.E.F.):	$v_A = 4u/9$, $v_B = 10u/9$	M1 A1		
	Find speed of B after striking barrier (ignore sign):	$v_B' = -ev_B$ $[= -10eu/9]$	M1		
	Use conservation of momentum, e.g.:	$2mw_A + mw_B = 2mv_A + m\,v_B'$	B1		
	Use restitution (must be consistent with prev. eqn.):	$w_A - w_B = -\frac{2}{3}(v_A - v_B')$	B1		
	Substitute and take $w_B = 5w_A$ *or* $-5w_A$:	$2w_A \pm 5w_A = 8u/9 - 10eu/9$			
	(M1 needs 2 eqns with $+5w_A$ *or* $-5w_A$):	$w_A - (\pm 5w_A) = -\frac{2}{3}(4u/9 + 10eu/9)$	M1		
	Find values of e (M1 for either):	$+5w_A \Rightarrow 2/13$, $-5w_A \Rightarrow \frac{1}{2}$	M1 A1, A1	11	**[11]**
6	State or find the mean of X:	$E(X) = 1/\frac{1}{3} = 3$	B1	1	
	Find $P(X > 8)$:	$P(X > 8) = (\frac{2}{3})^8$	M1		
		$= 256/6561$ *or* $0{\cdot}0390$	A1	2	
	Formulate condition for n (M0 here if equality used):	$1 - (\frac{2}{3})^{n-1} > 0{\cdot}99$, $(\frac{2}{3})^{n-1} < 0{\cdot}01$	M1		
	Take logs (any base) to give inequality for n:	$n - 1 > \log 0{\cdot}01 / \log \frac{2}{3}$	M1		
	Find $n_{\min}$:	$n - 1 > 11{\cdot}4$, $n_{\min} = 13$	A1	3	**[6]**
7	State hypotheses (A.E.F.):	H_0: $\mu = 7{\cdot}5$, H_1: $\mu < 7{\cdot}5$	B1		
	Calculate sample mean:	$\bar{x} = 70{\cdot}4 / 10 = 7{\cdot}04$	M1		
	Estimate population variance: (allow biased here: $0{\cdot}848$ *or* $0{\cdot}9209^2$)	$s^2 = 8{\cdot}48 / 9 = 211/225$ *or* $0{\cdot}9422$ *or* $0{\cdot}9707^2$	M1		
	Calculate value of t (to 3 sf):	$t = (\bar{x} - 7{\cdot}5)/(s/\sqrt{10}) = \pm 1{\cdot}49_{[9]}$	M1 *A1		
	State or use correct tabular t value: (or can compare $\bar{x}$ with $7{\cdot}5 - 0{\cdot}425 = 7{\cdot}07_{[5]}$)	$t_{9,\,0.9} = 1{\cdot}38_{[3]}$	*B1		
	correct conclusion (AEF, dep *A1, *B1):	Mean is less than 7·5	B1	7	**[7]**
8 (i)	Show that $A = \lambda$:	$\int_0^\infty A\mathrm{e}^{-\lambda t}\,\mathrm{d}t = \left[-(A/\lambda)\,\mathrm{e}^{-\lambda t}\right]_0^\infty$ $= A/\lambda = 1$ if $A = \lambda$ **A.G.**	M1 A1	2	
(ii)	Find estimate for λ:	$\int_0^1 \lambda\mathrm{e}^{-\lambda t}\,\mathrm{d}t$ *or* $\left[-\mathrm{e}^{-\lambda t}\right]_0^1 = 16/100$	M1		
		$1 - \mathrm{e}^{-\lambda} = 0{\cdot}16$			
		$\lambda = -\ln 0{\cdot}84 = 0{\cdot}174$	M1 A1		
	State or use eqn. for median m of T:	$\left[-\mathrm{e}^{-\lambda t}\right]_0^m = \frac{1}{2}$ (A.E.F.)	M1		
	Find value of m:	$\mathrm{e}^{-\lambda m} = \frac{1}{2}$, $m = \lambda^{-1}\ln 2 = 3{\cdot}98$	M1 A1	6	**[8]**

Question Number	Mark Scheme Details			Part Mark	To
9 (i)	Find sample coefficient using $r^2 = b_1 b_2$:	$r^2 = 4.21 \times 0.043 = 0.181$ *or* 0.425^2	M1 A1		
		$r = 0.425$	*A1	3	
(ii)	State both hypotheses:	H_0: $\rho = 0$, H_1: $\rho \neq 0$	B1		
	State or use correct tabular one-tail r value:	$r_{10,\ 5\%} = 0.549$	*B1		
	Valid method for reaching conclusion:	Accept H_0 if $\lvert r \rvert <$ tabular value	M1		
	Correct conclusion (AEF, dep *A1, *B1):	There is no non-zero correlation	A1	4	
(iii)	Solve regression eqns for mean values:	$\bar{x} = 7.72$ *and* $\bar{y} = 31.6$	M1 A1	2	
(iv)	Estimate x from either eqn:	$x = 6.46$ *or* 0.751	B1		
	State valid comment on reliability:	Not reliable because e.g. value of r is small *or* range of data is unknown *or* two estimates of x very different	B1	2	[11]
10	State (at least) null hypothesis (A.E.F.):	H_0: No difference in preferences [or independent]	B1		
	Find expected values (to 1 d.p.): (lose A1 if rounded to integers)	28·57 37·71 13·71 21·43 28·29 10·29	M1 A1		
	Calculate value of χ^2:	$\chi^2 = 0.411 + 0.078 + 0.214 + 0.549 + 0.104 + 0.286 = 1.64$ (*or* 1·61 if 1 d.p. used)	M1 A1		
	State or use correct tabular χ^2 value (to 3 s.f.):	$\chi_{2,\ 0.95}{}^2 = 5.99$[1]	B1		
	Valid method for reaching conclusion:	Accept H_0 if $\chi^2 \le$ tabular value	M1		
	Conclusion consistent with correct values (A.E.F):	No difference in preferences	A1	8	
	Calculate new value $\chi_{new}{}^2$ of χ^2:	$\chi_{new}{}^2 = n \times \chi^2$	M1		
	State or use correct tabular χ^2 value:	$\chi_{2,\ 0.95}{}^2 = 5.99$[1]	B1		
	Find n_{min}:	$n > 5.99/1.64$, $n_{min} = 4$	M1 A1	4	[12]
11a	Use conservation of energy (B0 for $v^2 = \ldots$):	$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mga(1 - \cos\theta)$ $[v^2 = u^2 + 2ga(1 - \cos\theta)]$	M1 A1		
	Equate radial forces (R may be taken as zero):	$R = mg\cos\theta - mv^2/a$	M1 A1		
	Eliminate v^2 to find $\cos\theta$ when $R = 0$:	$0 = mg(3\cos\theta - 2) - mu^2/a$			
	(denoting this θ by θ_1)	$\cos\theta_1 = \frac{1}{3}(2 + 2/5) = 4/5$ **A.G.**	M1 A1	6	
	Find v^2 at this point:	$v_1{}^2 = 2ga/5 + 2ga/5 = 4ga/5$	B1		
	Find vertical comp. v_2 of v at plane: (using u in place of v_1 can earn M1 A0) (A.E.F.)	$v_2{}^2 = v_1{}^2 \sin^2\theta_1 + 2ga(1 + \cos\theta_1)$ $= (4/5 \times 3^2/5^2 + 2 \times 9/5)ga$	M1		
		$= 486ga/125$ *or* $3.89ga$	A1		
	Find vertical comp. v_3 of rebound speed:	$v_3 = (5/9)\, v_2$ $[v_3{}^2 = 6ga/5]$	M1		
	Find vertical height reached:	$v_3{}^2/2g = (5/9)^2 (486ga/125) / 2g$	M1		
		$= 3a/5$	A1	6	[12]

Question Number	Mark Scheme Details			Part Mark	To
11b	State hypotheses:	H_0: $\mu_X = \mu_Y$, H_1: $\mu_X < \mu_Y$	B1		
	Estimate population variance using X's sample: allow use of biased: $\sigma_{X,60}{}^2 = 0{\cdot}4891$ *or* $0{\cdot}6994^2$)	$s_X{}^2 = (85{\cdot}8 - 58{\cdot}2^2/60) / 59$ [= $0{\cdot}4974$ *or* $0{\cdot}7053^2$]	M1 A1		
	Estimate population variance using Y's sample: (allow use of biased: $\sigma_{Y,80}{}^2 = 0{\cdot}8691$ *or* $0{\cdot}9323^2$)	$s_Y{}^2 = (188{\cdot}6 - 97{\cdot}6^2/80) / 79$ [= $0{\cdot}8801$ *or* $0{\cdot}9381^2$]	M1 A1		
	Estimate population variance for combined sample: (allow use of $\sigma_{X,60}{}^2$, $\sigma_{Y,80}{}^2$)	$s^2 = s_X{}^2/60 + s_Y{}^2/80$ $= 0{\cdot}01929$ *or* $0{\cdot}1389^2$ (*or* $0{\cdot}01901$ *or* $0{\cdot}1379^2$)	M1 A1		
	Calculate value of z (to 2 d.p., either sign):	$z = (0{\cdot}97 - 1{\cdot}22) / s$ $= -0{\cdot}25/0{\cdot}1389 = -1{\cdot}80$	M1		
		(*or* $-1{\cdot}813$)	A1		
	Find $\Phi(z)$:	$\Phi(z) = 0{\cdot}9641$ (*or* $0{\cdot}9651$)	M1		
	Find corresponding significance level:	$3{\cdot}59$ (*or* $3{\cdot}49$)	M1		
	Find set of possible values of α (to 1 d.p.):	$\alpha > 3{\cdot}6$	A1	12	
	S.R. Allow (implicit) assumption of equal variances as follows, but deduct A1 if not explicit:				
	Find pooled estimate of common variance s^2:	$(60\sigma_{X,60}{}^2 + 80\sigma_{Y,80}{}^2)/138$ $= 0{\cdot}7165$ *or* $0{\cdot}8465^2$			
	Calculate value of z (to 2 d.p.):	$z = (0{\cdot}97 - 1{\cdot}22)/s\sqrt{(1/60+1/80)}$ $= -1{\cdot}73$			
	Find set of possible values of α (to 1 d.p.):	$\Phi(z) = 0{\cdot}9582$, $\alpha > 4{\cdot}2$			**[12]**