

Cambridge Assessment International Education Cambridge International Advanced Level

#### **FURTHER MATHEMATICS**

9231/13 October/November 2019

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Paper 1 MARK SCHEME Maximum Mark: 100

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE<sup>™</sup>, Cambridge International A and AS Level components and some Cambridge O Level components.

#### **Generic Marking Principles**

October/Novembe "". Mymathscioud.com These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a guestion. Each guestion paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:** 

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the guestion •
- the specific skills defined in the mark scheme or in the generic level descriptors for the guestion .
- the standard of response required by a candidate as exemplified by the standardisation scripts. •

GENERIC MARKING PRINCIPLE 2.

Marks awarded are always whole marks (not half marks, or other fractions).

#### **GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do •
- marks are not deducted for errors •
- marks are not deducted for omissions •
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the • question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:** 

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:** 

Marks should be awarded using the full range of marks defined in the mark scheme for the guestion (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:** 

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

#### Mark Scheme Notes

October/Novembe "".mymainscioud.com The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

#### Types of mark

- М Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- Α Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- В Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically DM or DB says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
  - Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B FT marks are given for correct work only.

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#### Abbreviations

- AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
- October/Novembe "N. Mymainscioud.com AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
- CWO **Correct Working Only**
- ISW Ignore Subsequent Working
- SOI Seen Or Implied
- SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
- WWW Without Wrong Working
- AWRT Answer Which Rounds To

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October/Novembe Myn

Question	Answer	Marks	Guidance	SSC/O
1	$A = \int_0^1 x^a dx = \left[\frac{x^{a+1}}{a+1}\right]_0^1 = \frac{1}{a+1}$	B1	Finds area of region.	-Ud.com
	$A\overline{x} = \int_{0}^{1} xy  dx = \int_{0}^{1} x^{a+1}  dx = \left[\frac{x^{a+2}}{a+2}\right]_{0}^{1} = \frac{1}{a+2}$	M1 A1	Finds $\int_{0}^{1} xy  dx$ .	
	$2A\overline{y} = \int_{0}^{1} y^{2} dx = \int_{0}^{1} x^{2a} dx = \left[\frac{x^{2a+1}}{2a+1}\right]_{0}^{1} = \frac{1}{2a+1}$	M1 A1	Finds $\int_{0}^{1} y^2 dx$ .	
	$(\overline{x}, \overline{y}) = \left(\frac{a+1}{a+2}, \frac{a+1}{2(2a+1)}\right)$	A1	Both coordinates correct.	
		6		

#### Cambridge International A Level – Mark Scheme PUBLISHED

October/Novembe myn

Question	Answer	Marks	Guidance	Scio,
2	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a}{ax+1} = (-1)^0 \frac{0!a^1}{(ax+1)^1}$ so true for $n = 1$ .	M1 A1	Proves base case.	-Ud.Com
	Assume that $\frac{d^{k} y}{dx^{k}} = (-1)^{k-1} \frac{(k-1)! a^{k}}{(ax+1)^{k}}$ for some positive integer k.	B1	States inductive hypothesis.	
	Then $\frac{d^{k+1}y}{dx^{k+1}} = -ka(-1)^{k-1} \frac{(k-1)!a^k}{(ax+1)^{k+1}} = (-1)^k \frac{k!a^{k+1}}{(ax+1)^{k+1}}$ so true for $n = k+1$ .	M1 A1	Differentiates k <sup>th</sup> derivative.	
	By induction, true for every positive integer $n$ .	A1	States conclusion.	
		6		

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October/Novembe Mynyns

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Question	Answer	Marks	Guidance	ASCIO.
3(i)	$I_{n+2} = \left[\frac{x^{-n-1}}{-n-1}\sin \pi x\right]_{\frac{1}{2}}^{1} - \pi \int_{\frac{1}{2}}^{1} \frac{x^{-n-1}}{-n-1}\cos \pi x  dx$	M1 A1	Integrates by parts.	-ud.com
	$=\frac{2^{n+1}}{n+1} + \frac{\pi}{n+1} \left( \left[ \frac{x^{-n}}{-n} \cos \pi x \right]_{\frac{1}{2}}^{1} + \pi \int_{\frac{1}{2}}^{1} \frac{x^{-n}}{-n} \sin \pi x  dx \right)$	M1	Integrates by parts again.	
	$=\frac{2^{n+1}}{n+1} + \frac{\pi}{n+1} \left( \frac{1}{n} - \frac{\pi}{n} I_n \right) $ (1)	M1	Uses $I_n$ .	
	$\Rightarrow (n+1)I_{n+2} = 2^{n+1} + \pi \left(\frac{1}{n} - \frac{\pi}{n}I_n\right)$			
	$\Rightarrow n(n+1)I_{n+2} = 2^{n+1}n + \pi - \pi^2 I_n$	A1	AG	
		5		
3(ii)	$2I_3 = 4 + \pi - \pi^2 I_1$	M1	Substitutes $I_3$ into reduction formula.	
	$12I_5 = 48 + \pi - \frac{\pi^2}{2} \left(4 + \pi - \pi^2 I_1\right)$			
	$\Rightarrow I_5 = 4 + \frac{1}{24} \left( 2\pi - 4\pi^2 - \pi^3 + \pi^4 I_1 \right)$	A1	AEF, must be exact with fractions simplified.	
		2		

# Cambridge International A Level – Mark Scheme **PUBLISHED**

October/Novembe myn

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Question	Answer	Marks	Guidance	SSCIO,
4(i)	$x^{2} + 1 = (ax + b)(2x + 1) + c$	M1	Uses that $2x + 1$ is the quotient.	Ud.con
	$\Rightarrow a = \frac{1}{2}, b = -\frac{1}{4}$	A1 A1		
		3		
4(ii)	$x = \frac{1}{2}$	B1 FT		
		1		

#### Cambridge International A Level – Mark Scheme **PUBLISHED**

October/Novembe nym

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Question	Answer	Marks	Guidance	SCIO, S
4(iii)	10	B1	Intersection (0,-4) given and asymptotes drawn.	Ud.con
		B1	Left branch correct.	
	10	B1 FT	Right branch correct.	
	-10 -5 -0 -5 10		Deduct at most one mark for poor forms at infinity.	
		3		

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October/Novembe Myn

Question	Answer	Marks	Guidance	ns
5(i)	$\sum_{r=1}^{N} (5r+1)(5r+6) = 25 \sum_{r=1}^{N} r^2 + 35 \sum_{r=1}^{N} r + 6N$	M1	Expands.	td.com
	$25\left(\frac{1}{6}N(N+1)(2N+1)\right) + 35\left(\frac{1}{2}N(N+1)\right) + 6N$	M1	Substitutes formulae for $\sum r$ and $\sum r^2$ .	
	$= N\left(\frac{25}{6}\left(2N^{2}+3N+1\right)+\frac{35}{2}N+\frac{35}{2}+6\right) = \frac{1}{3}N\left(25N^{2}+90N+83\right)$	A1	Simplifies to the given answer (AG).	
		3		
5(ii)	$\frac{1}{(5r+1)(5r+6)} = \frac{1}{5} \left( \frac{1}{5r+1} - \frac{1}{5r+6} \right)$	M1 A1	Finds partial fractions.	
	$T_N = \frac{1}{5} \left( \frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \dots + \frac{1}{5N+1} - \frac{1}{5N+6} \right)$	M1	Expresses terms as differences.	
	$\left[\frac{1}{5}\left(\frac{1}{6} - \frac{1}{5N+6}\right) = \frac{1}{30} - \frac{1}{5(5N+6)}\right]$	A1	At least 3 terms including last.	
		4		
5(iii)	$\frac{S_N}{N^3}T_N \rightarrow \frac{25}{3} \times \frac{1}{30} = \frac{5}{18}$	M1 A1	Divides $S_N$ by $N^3$ and takes limits as $N \to \infty$	
		2		

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October/Novembe myn

Question	Answer	Marks	Guidance	SSC/O
6(i)	$\overrightarrow{AB} = \begin{pmatrix} 1\\2\\7 \end{pmatrix}$	B1		UC.COM
	$\overrightarrow{OC} \times \overrightarrow{AB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 1 & 2 & 7 \end{vmatrix} = \begin{pmatrix} -9 \\ -6 \\ 3 \end{pmatrix} = t \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$	M1 A1	Finds direction of common perpendicular.	
	$\boxed{\begin{array}{c} \begin{pmatrix} 1\\-1\\0 \end{pmatrix} \begin{pmatrix} 3\\2\\-1 \end{pmatrix}}{\sqrt{3^2 + 2^2 + 1^2}} = \frac{1}{\sqrt{14}} = 0.267$	M1 A1	Uses formula for shortest distance.	
		5		
6(ii)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 1 \\ 3 & 2 & -1 \end{vmatrix} = t \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix}$	M1 A1	Finds normal to plane.	
	-(0)+4(0)+5(0)=0	M1	Uses point on plane.	
	-x + 4y + 5z = 0	A1	AEF	
		4		

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October/Novembe nym

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Question	Answer	Marks	Guidance Str.scio	75
7(i)	$\sqrt{-7y}(-7y) + 2(-7y) + \sqrt{-7y} + 7 = 0$	M1	Uses given substitution and eliminates radical.	ld.con
	$\Rightarrow \sqrt{-7y} \left(-7y+1\right) = 14y-7 \Rightarrow -7y \left(-7y+1\right)^2 = \left(14y-7\right)^2$			
	$\Rightarrow 49y^3 + 14y^2 - 27y + 7 = 0$	A1	AG	
	$y = \frac{x^2}{-7} = \frac{x^2}{\alpha\beta\gamma}$	M1	Uses $\alpha\beta\gamma = -7$ .	
	So roots are $\frac{\alpha^2}{\alpha\beta\gamma} = \frac{\alpha}{\beta\gamma}, \ \frac{\beta^2}{\alpha\beta\gamma} = \frac{\beta}{\alpha\gamma}, \ \frac{\gamma^2}{\alpha\beta\gamma} = \frac{\gamma}{\alpha\beta}$	A1	AG	
		4		
7(ii)	$\frac{\alpha}{\beta\gamma} + \frac{\beta}{\alpha\gamma} + \frac{\gamma}{\alpha\beta} = -\frac{2}{7}, \ \frac{1}{\gamma^2} + \frac{1}{\beta^2} + \frac{1}{\alpha^2} = -\frac{27}{49}$	B1	States sum of roots and $\alpha'\beta' + \alpha'\gamma' + \beta'\gamma'$ .	
	$\frac{\alpha^{2}}{\beta^{2}\gamma^{2}} + \frac{\beta^{2}}{\alpha^{2}\gamma^{2}} + \frac{\gamma^{2}}{\alpha^{2}\beta^{2}} = \left(-\frac{2}{7}\right)^{2} - 2\left(-\frac{27}{49}\right) = \frac{58}{49}$	M1 A1	Uses $\alpha'^{2} + \beta'^{2} + \gamma'^{2} = (\alpha' + \beta' + \gamma')^{2} - 2(\alpha'\beta' + \alpha'\gamma' + \beta'\gamma')$ AG	
		3		
7(iii)	$49\left(\frac{\alpha^3}{\beta^3\gamma^3} + \frac{\beta^3}{\alpha^3\gamma^3} + \frac{\gamma^3}{\alpha^3\beta^3}\right) = -14\left(\frac{58}{49}\right) + 27\left(-\frac{2}{7}\right) - 21$	M1	Uses $49\alpha'^3 = -14\alpha'^2 + 27\alpha' - 7$ .	
	$\Rightarrow \frac{\alpha^3}{\beta^3 \gamma^3} + \frac{\beta^3}{\alpha^3 \gamma^3} + \frac{\gamma^3}{\alpha^3 \beta^3} = -\frac{317}{343}$	A1		
		2		

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Question	Answer	Marks	Guidance	.75
8(i)	Eigenvalues of (upper diagonal matrix) <b>A</b> are 2, <i>m</i> and 1. (Or from characteristic equation: $(\lambda - 2)(\lambda - m)(\lambda - 1) = 0$ )	B1		Ud.com
	$\lambda = 2: \mathbf{e}_{1} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & m-2 & 1 \\ 0 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 2-m \\ 0 \\ 0 \end{pmatrix} = t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	M1 A1	Uses vector product (or equations) to find corresponding eigenvectors.	
	$\lambda = m: \mathbf{e}_{2} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 - m & m & 1 \\ 0 & 0 & 7 \end{vmatrix} = \begin{pmatrix} 7m \\ 7(m-2) \\ 0 \end{pmatrix} = t \begin{pmatrix} m \\ m-2 \\ 0 \end{pmatrix}$	A1		
	$\lambda = 1: \mathbf{e}_{3} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & m & 1 \\ 0 & m-1 & 7 \end{vmatrix} = \begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix} = t \begin{pmatrix} 6m+1 \\ -7 \\ m-1 \end{pmatrix}$	A1		
	Thus $\mathbf{P} = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & 1 \end{pmatrix}$	M1 A1 FT	Or correctly matched permutations of columns. No follow through on two or more zero eigenvectors.	
		7		

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October/Novembe myn

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Question	Answer	Marks	Guidance	ns SC/O
8(ii)	$\mathbf{M}^{7}\mathbf{P} = \mathbf{P}\mathbf{D}^{7}\mathbf{P}^{-1}\mathbf{P} = \mathbf{P}\mathbf{D}^{7} = \begin{pmatrix} 1 & m & 6m+1 \\ 0 & m-2 & -7 \\ 0 & 0 & m-1 \end{pmatrix} \begin{pmatrix} 2^{7} & 0 & 0 \\ 0 & m^{7} & 0 \\ 0 & 0 & 1 \end{pmatrix}$	M1 A1 FT	Applies $\mathbf{M}^7 = \mathbf{P}\mathbf{D}^7\mathbf{P}^{-1}$ .	UD.COM
	$= \begin{pmatrix} 2^7 & m^8 & 6m+1 \\ 0 & m^8 - 2m^7 & -7 \\ 0 & 0 & m-1 \end{pmatrix}$	A1	Order of columns might be swapped depending on <b>P</b> .	
		3		

Question	Answer	Marks	Guidance
9(i)	Write $c = \cos \theta$ , $s = \sin \theta$ . $\cos 6\theta + i \sin 6\theta = (c + is)^{6}$	M1	Uses binomial theorem.
	$\Rightarrow \cos 6\theta = c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1	
	$c^{6} - 15c^{4}s^{2} + 15c^{2}s^{4} - s^{6} = c^{6} - 15c^{4}(1 - c^{2}) + 15c^{2}(1 - c^{2})^{2} - (1 - c^{2})^{3}$	M1	Uses $c^2 = 1 - s^2$ .
	$= c^{6} - 15c^{4}(1 - c^{2}) + 15c^{2}(1 - 2c^{2} + c^{4}) - (1 - 3c^{2} + 3c^{4} - c^{6})$	A1	
	$= 32c^6 - 48c^4 + 18c^2 - 1$	M1	Divides numerator and denominator by $c^6$ .
	$\Rightarrow \sec 6\theta = \frac{1}{32c^{6} - 48c^{4} + 18c^{2} - 1} = \frac{\sec^{6} \theta}{32 - 48\sec^{2} \theta + 18\sec^{4} \theta - \sec^{6} \theta}$	A1	AG
		6	

#### Cambridge International A Level – Mark Scheme PUBLISHED

October/Novembe Myn

Question	Answer	Marks	Guidance	?/ <sub>0</sub> ,
9(ii)	$x^{6} = 2(32 - 48x^{2} + 18x^{4} - x^{6}) \Longrightarrow \frac{x^{6}}{32 - 48x^{2} + 18x^{4} - x^{6}} = 2$	M1 A1	Relates with equation in part (i).	-Vd. Co,
	$\sec 6\theta = 2 \Longrightarrow \cos 6\theta = \frac{1}{2}$	M1	Solves $\cos 6\theta = \frac{1}{2}$ .	
	$x = \sec\frac{\pi}{18}$	A1	Gives one correct solution.	
	$x = \sec q\pi$ , $q = \frac{5}{18}, \frac{7}{18}, \frac{11}{18}, \frac{13}{18}, \frac{17}{18}$	A1	Gives five other solutions. Allow different values of $q$ as long as all six solutions are found.	
		5		

October/Novembe nym

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Question	Answer	Marks	Guidance
10(i)	$ \begin{pmatrix} 1 & 5 & 1 \\ 1 & -2 & -2 \\ 2 & 3 & \theta \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5 & 1 \\ 0 & -7 & -3 \\ 0 & 0 & \theta + 1 \end{pmatrix} $	M1 A1	Reduces to echelon form. At least one row operation for M1.
	$r(\mathbf{A}) = 3$ if $\theta \neq -1$	A1	
	$r(\mathbf{A}) = 2$ if $\theta = -1$	B1	
		4	
10(ii)	x +5y +z = -1 -7y -3z = 1 $(\theta+1)z = (\theta+1)$	M1	Uses reduced form of augmented matrix or eliminates variables from scratch.
	$z = 1, y = -\frac{4}{7}, x = \frac{6}{7}$	A1 A1	One correct. All three correct.
		3	
10(iii)	x +5y +z = -1 -7y -3z = 1 $(\theta+1)z = (\theta+1)$		
	z = t	M1	Uses parameter.
	$y = -\frac{3t+1}{7}, x = \frac{8t-2}{7}$	A1 A1	
		3	

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Question	Answer	Marks	Guidance	SSCIO,
10(iv)	$x+5y+z=-1,-7y -3z=1,(\theta+1)z=\phi+1$	M1	Uses reduced form of augmented matrix or eliminates variables from scratch	Ud.com
	$\theta = -1 \Rightarrow \phi = -1$ so no solution (inconsistent).	A1		
		2		

Question	Answer	Marks	Guidance
11E(i)	$w = \cos y \Rightarrow \frac{\mathrm{d}w}{\mathrm{d}x} = -\sin y \frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
	$\frac{d^2w}{dx^2} = -\sin y \frac{d^2 y}{dx^2} - \cos y \left(\frac{dy}{dx}\right)^2$	B1	
	$\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} + 2\frac{\mathrm{d}w}{\mathrm{d}x} + w = -\sin y \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - \cos y \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 - 2\sin y \frac{\mathrm{d}y}{\mathrm{d}x} + \cos y$	M1	Uses substitution to obtain $w - x$ equation, AG.
	$= -\cos y \left( e^{-2x} \sec y \right) = -e^{-2x}$	A1	
		4	

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October/Novembe Myn

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Question	Answer	Marks	Guidance	.ns
11E(ii)	$m^2 + 2m + 1 = 0 \Longrightarrow m = -1$	M1	Finds CF.	Ud.Com
	CF: $w = (Ax + B)e^{-x}$	A1		
	PI: $w = ke^{-2x} \Longrightarrow w' = -2ke^{-2x} \Longrightarrow w'' = 4ke^{-2x}$	M1	Forms PI and differentiates.	
	$4k - 4k + k = -1 \Longrightarrow k = -1$	A1		
	$w = (Ax + B)e^{-x} - e^{-2x}$	A1	States general solution.	
	$x=0, y=\frac{1}{3}\pi, w=\frac{1}{2} \Longrightarrow B=\frac{3}{2}$	B1	Uses initial conditions to find constants.	
	$w' = -(Ax + B)e^{-x} + Ae^{-x} + 2e^{-2x}$	M1	Differentiates general solution.	
	$x = 0, y = \frac{1}{3}\pi, y' = \frac{\sqrt{3}}{3}, w' = -\frac{1}{2} \Longrightarrow -\frac{1}{2} = -\frac{3}{2} + A + 2 \Longrightarrow A = -1$	M1 A1	Substitutes initial conditions.	
	$y = \cos^{-1}\left(\left(\frac{3}{2} - x\right)e^{-x} - e^{-2x}\right)$	A1	States particular solution for $y$ in terms of $x$ .	
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Question	Answer	Marks	Guidance	SC/O
11O(i)	$e^{2\alpha} - e^{-2\alpha} = 2(e^{\alpha} + e^{-\alpha}) \Longrightarrow e^{\alpha} - e^{-\alpha} = 2$	M1	Sets equations equal and divides by $e^{\alpha} + e^{-\alpha}$ .	Ud.Com
	$e^{2\alpha} - 2e^{\alpha} - 1 = 0 \Longrightarrow e^{\alpha} = 1 + \sqrt{2}$	M1 A1	Forms quadratic in $e^{\alpha}$ , AG.	
	$\alpha = \ln\left(1 + \sqrt{2}\right)$	A1	Must be exact.	
	$r = 2\left(1 + \sqrt{2} + \sqrt{2} - 1\right) = 4\sqrt{2}$	M1 A1	Substitutes to find <i>r</i> .	
		6		
110(ii)		B1	$C_1$ has correct shape.	
		B1	$C_2$ has correct shape.	
		B1	Intersection points positioned correctly.	
		3		

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October/Novembe myma

Question	Answer	Marks	Guidance	SCIO,
11O(iii)	$2\int_{0}^{\ln(1+\sqrt{2})} (e^{\theta} + e^{-\theta})^{2} d\theta - \frac{1}{2}\int_{0}^{\ln(1+\sqrt{2})} (e^{2\theta} - e^{-2\theta})^{2} d\theta$ $= \int_{0}^{\ln(1+\sqrt{2})} 5 + 2e^{2\theta} + 2e^{-2\theta} - \frac{1}{2}e^{4\theta} - \frac{1}{2}e^{-4\theta} d\theta$	M1 A1	Uses $\frac{1}{2}\int r^2 d\theta$ to formulate correct area.	u.com
	$= \left[ 5\theta + e^{2\theta} - e^{-2\theta} - \frac{1}{8}e^{4\theta} + \frac{1}{8}e^{-4\theta} \right]_{0}^{\ln(1+\sqrt{2})}$	M1 A1	Expands and integrates.	
	$=5\ln(1+\sqrt{2})+\left(1+\sqrt{2}\right)^{2}-\left(1+\sqrt{2}\right)^{-2}-\frac{1}{8}\left(\left(1+\sqrt{2}\right)^{4}-\left(1+\sqrt{2}\right)^{-4}\right)=5.82$	A1		
		5		