Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **May/June 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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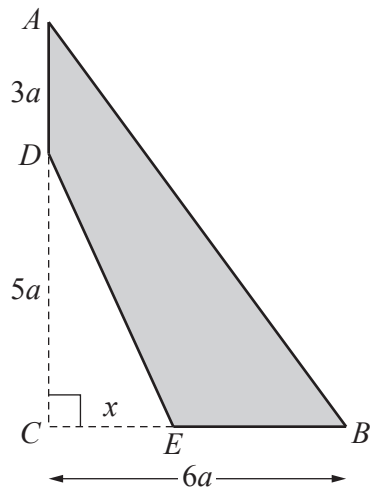
1 A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held at the point A, where OA makes an angle α with the downward vertical through O, and with the string taut. The particle P is projected perpendicular to OA in an upwards direction with speed $\sqrt{3ag}$. It then starts to move along a circular path in a vertical plane. The string goes slack when P is at B, where OB makes an angle θ with the upward vertical.

Given that $\cos\alpha = \frac{4}{5}$, find the value of $\cos\theta$. [4]

2 One end of a light elastic string, of natural length a and modulus of elasticity λmg, is attached to a fixed point O. The string lies on a smooth horizontal surface. A particle P of mass m is attached to the other end of the string. The particle P is projected in the direction OP. When the length of the string is $\frac{4}{3}a$, the speed of P is $\sqrt{2ag}$. When the length of the string is $\frac{5}{3}a$, the speed of P is $\frac{1}{2}\sqrt{2ag}$.

Find the value of λ. [4]

3



A uniform lamina is in the form of a triangle ABC, with $AC = 8a$, $BC = 6a$ and angle $ACB = 90°$. The point D on AC is such that $AD = 3a$. The point E on CB is such that $CE = x$ (see diagram). The triangle CDE is removed from the lamina.

(a) Find, in terms of a and x, the distance of the centre of mass of the remaining object $ADEB$ from AC. [4]

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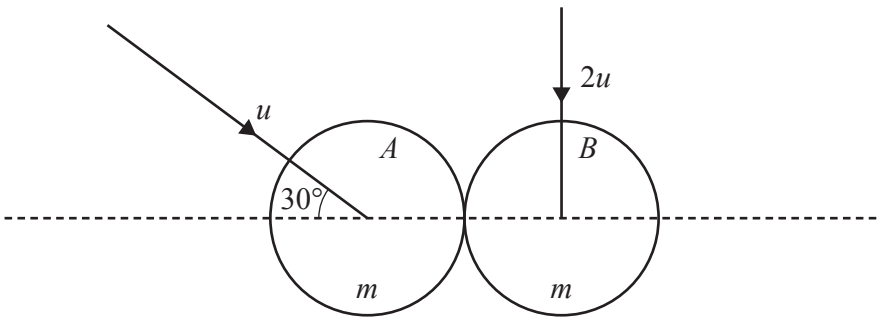
The object $ADEB$ is on the point of toppling about the point E when the object is in the vertical plane with its edge EB on a smooth horizontal surface.

(b) Find x in terms of a. [3]

4



Two identical smooth uniform spheres A and B each have mass m. The two spheres are moving on a smooth horizontal surface when they collide with speeds u and $2u$ respectively. Immediately before the collision, A's direction of motion makes an angle of 30° with the line of centres, and B's direction of motion is perpendicular to the line of centres (see diagram). After the collision, A and B are moving in the same direction. The coefficient of restitution between the spheres is e.

(a) Find the value of e. [5]

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(b) Find the loss in the total kinetic energy of the spheres as a result of the collision. [3]

5 One end of a light elastic string, of natural length $12a$ and modulus of elasticity kmg, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves with constant speed $\frac{3}{2}\sqrt{3ag}$ in a horizontal circle with centre at a distance $12a$ below O. The string is inclined at an angle θ to the downward vertical through O.

(a) Find, in terms of a, the extension of the string. [5]

(b) Find the value of k. [3]

6 A particle of mass m kg falls vertically under gravity, from rest. At time t s, P has fallen x m and has velocity $v\,\mathrm{m\,s^{-1}}$. The only forces acting on P are its weight and a resistance of magnitude $kmgv$ N, where k is a constant.

(a) Find an expression for v in terms of t, g and k. [5]

(b) Given that $k = 0.05$, find, in metres, how far P has fallen when its speed is $12\,\mathrm{m\,s^{-1}}$. [5]

7 The points O and P are on a horizontal plane, a distance 8 m apart. A ball is thrown from O with speed $u\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At the same instant, a model aircraft is launched with speed $5\,\mathrm{m\,s^{-1}}$ parallel to the horizontal plane from a point 4 m vertically above P. The model aircraft moves in the same vertical plane as the ball and in the same horizontal direction as the ball. The model aircraft moves horizontally with a constant speed of $5\,\mathrm{m\,s^{-1}}$. After T s, the ball and the model aircraft collide.

(a) Find the value of T. [6]

(b) Find the direction in which the ball is moving immediately before the collision. [3]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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