



Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

8890732124

FURTHER MATHEMATICS

9231/13

Paper 1 Further Pure Mathematics 1

October/November 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Any blank pages are indicated.

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1 It is given that

$\alpha + \beta + \gamma = 3$, $\alpha + \beta + \gamma = 3$, $\alpha + \beta + \gamma = 0$	$\alpha + \beta + \gamma = 3$,	$\alpha^2 + \beta^2 + \gamma^2 = 5,$	$\alpha^3 + \beta^3 + \gamma^3 = 6.$
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The cubic equation $x^3 + bx^2 + cx + d = 0$ has roots α , β , γ . Find the values of b, c and d. [6]

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(a)	Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n} r(r+1)(r+2)$ in terms fully factorising your answer.

		$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}.$	
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Г	Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+1)}$	$\overline{r+2)}$.	
	r=1	,	

3 The sequence of real numbers a_1 , a_2 , a_3 , ... is such that $a_1 = 1$ and

$$a_{n+1} = \left(a_n + \frac{1}{a_n}\right)^3.$$

(a) Prove by mathematical induction that $\ln a_n \ge 3^{n-1} \ln 2$ for all integers $n \ge 2$. [6] [You may use the fact that $\ln\left(x+\frac{1}{x}\right) > \ln x$ for x > 0.] **(b)** Show that $\ln a_{n+1} - \ln a_n > 3^{n-1} \ln 4$ for $n \ge 2$. [2] 4 The matrix **M** is given by $\mathbf{M} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$.

(a)	The matrix M represents a sequence of two geometrical transformations.	
	State the type of each transformation, and make clear the order in which they are applied.	2]
		•••
		•••
		•••
(b)	Find the values of θ , for $0 \le \theta \le \pi$, for which the transformation represented by M has exact one invariant line through the origin, giving your answers in terms of π .	 ly 9]
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plane Π has equation $\mathbf{r} = -2\mathbf{i} + 3\mathbf{j} + 3\mathbf{l}$		nnu nymatr
Find a Cartesian equation of Π , giving	g your answer in the form $ax + by + cz = d$.	[4]
		•••••
		••••••
ine l passes through the point P with p	osition vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ and is parallel to the	ne vector k .
ine l passes through the point P with p		ne vector k.
Find the position vector of the point when the position vector of the position vector of the point when the position vector of the pos		[3]
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Find the position vector of the point w	here <i>l</i> meets Π.	[3]

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F	Find the perpendicular distance from P to Π .
	Find the perpendicular distance from P to Π .
-	

6

curve C has po	olar equation $r =$	$2\cos\theta(1+\sin\theta)$	(θ) , for $0 \le \theta$	$\leq \frac{1}{2}\pi$.	NW. MY MATT
Find the polar	coordinates of the	ne point on C to	hat is furthest	from the pole.	[5]
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(b)	Sketch C.	[2]	JOHO, COL

Find the area of the region bounded by C and the initial line, giving your answer in exact form. [6]

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7 The curve C has equation $y = \frac{4x+5}{4-4x^2}$.

(a)	Find the equations of the asymptotes of C.	[2]
(b)	Find the coordinates of any stationary points on C .	[4]

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[3]	OHOCO

(c) Sketch C, stating the coordinates of the intersections with the axes.

(d) Sketch the curve with equation $y = \left| \frac{4x+5}{4-4x^2} \right|$ and find in exact form the set of values of x for which $4|4x+5| > 5|4-4x^2|$. [6]

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Additional Page

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