AQA

A-LEVEL
Mathematics

MS03

Statistics 3

Mark scheme

June 2019

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as $13/47$ given as $13:47$ or $13:34$

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

GN8 Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.

Q	Solution	Marks	Total	Comments
1 (a)	95% $\Rightarrow$ z = **<u>1.96</u>**	B1		AWRT (1.95996)
	CI for $\mu_E - \mu_W$: $(\bar{e} - \bar{w}) \pm z\sqrt{\frac{s_E^2}{n_E} + \frac{s_W^2}{n_W}}$	M1 m1		General form used Correct form used for SD Accept pooling
	$(42.6 - 39.7) \pm 1.96\sqrt{\frac{6.2^2}{50} + \frac{5.3^2}{50}}$	AF1		F on z Pooling gives $1.96\sqrt{1.3306}$
	<u>2.9 ± 2.3 or (0.6, 5.2)</u>	A1	**5**	AWRT (2.261)
(b) (i)	**Random**	B1	**1**	CAO
(ii)	**Large samples** (both > 25 or 30) so can apply **Central Limit Theorem** to distributions of sample **means**	B1 Bdep1	**2**	Dependent on B1
		Total	**8**	

Q	Solution	Marks	Total	Comments
2 (a)(i)	$P(S \cap U) = 0.15 \times 0.10 = \underline{\mathbf{0.015}}$	B1	**1**	CAO
(ii)	$P(U) = (0.40 \times 0.15) + (0.45 \times 0.05) + (0.15 \times 0.10)$ or (i) $= 0.06 + 0.0225 + 0.015 = \underline{\mathbf{0.097 \text{ to } 0.098}}$	M1 A1	**2**	≥2 terms correct; may be implied AWFW (0.0975)
(iii)	$P(D \mid U) = \frac{P(D \cap U)}{P(U)} = \frac{0.40 \times 0.15}{(ii)}$ $= \frac{0.06}{0.0975} = \underline{\mathbf{0.612 \text{ to } 0.619}}$	M1 A1	**2**	May be implied AWFW (0.61538)
(iv)	$P(S \mid O) = \frac{0.15 \times (1 - 0.10)}{1 - (ii)} = \frac{0.135}{0.9025}$ $= \underline{\mathbf{0.149 \text{ to } 0.15}}$	M1 M1 A1	**3**	Numerator Denominator AWFW (0.14958)
(b)	$P(D \cap T \cap S \mid O)$ $= \frac{0.40 \times 0.85}{1 - (ii)} \times \frac{0.45 \times 0.95}{1 - (ii)} \times (iv) \times 3!$ $= \frac{0.34 \times 0.4275 \times 0.135 \times 6}{0.9025^3}$ **or** $= 0.3767 \times 0.4737 \times 0.1496 \times 6$ $= \underline{\mathbf{0.16}}$	M1 M1 M1 A1	**4**	≥2 terms correct in numerator $(1 - (ii))$ in denominator 3! or 6 or 3 AWRT (0.16016)
		Total	**12**	

Q	Solution	Marks	Total	Comments
3 (a)	$\hat{p} = \frac{68}{125}$ = **0.544**	B1		CAO; or equivalent
	98% $\Rightarrow$ z = **2.32 to 2.33**	B1		AWFW (2.3263)
	Approximate CI for p is:			
	$0.544 \pm 2.3263 \times \sqrt{\frac{0.544 \times 0.456}{125}}$	M1		Correct form of $\hat{p} \pm z\sqrt{\hat{p}(1-\hat{p})/125}$
		AF1		F on $\hat{p}$ and z
	0.544 ± (0.103 to 0.104)	A1		CAO/AWFW
	or			May be implied by **correct** answer
	(0.44 to 0.441, 0.647 to 0.65)			AWFW
	(44%, 65%)	A1	**6**	AWRT
(b)	Require $2 \times 2.3263 \times \sqrt{\frac{p(1-p)}{n}} \leq 0.1$ (10%)	M1		Allow 'no 2' and FT on CI from (a)
		A1		Allow p = 0.44 to 0.65
	–			
	Thus $\sqrt{n}$ = 22.1 to 23.3	M1		Attempted solution for $\sqrt{n}$ or n
	***n* = 490, 495, ..., 545**	A1	**4**	Must be to 'nearest 5'
		Total	**10**	

Q	Solution	Marks	Total	Comments
4	H_0: $\lambda_B = \lambda_A$ H_1: $\lambda_B > \lambda_A$	B1		Both
	5% $\Rightarrow$ $z =$ **1.64 to 1.65**			AWFW (1.64485)
	or	B1		
	p-value of z-calculated = **0.021 to 0.024 < 0.05**			AWFW (0.21556 or 0.02396)
	$\hat{\lambda}_A = \frac{315}{30} =$ **10.5** and $\hat{\lambda}_B = \frac{720}{60} =$ **12.0**	B1		Both CAO $\hat{\lambda} = \frac{1035}{90} =$ **11.5**
	$z = \frac{12.0 - 10.5}{\sqrt{\frac{12.0}{60} + \frac{10.5}{30}}} =$ **2.02**	M1 M1 Adep1		Correct numerator Correct denominator AWRT; dep on M1 M1 (2.02260)
	or			
	$z = \frac{12.0 - 10.5}{\sqrt{11.5\left(\frac{1}{60} + \frac{1}{30}\right)}} =$ **1.98**	(M1) (M1) (A1)		Correct numerator Correct denominator AWRT; dep on M1 M1 (1.97814)
	Thus evidence, at 5% level, to **support the claim** that $\lambda_B > \lambda_A$	Adep1	**7**	Dep on at least one M1 scored, z-value and CV
		Total	**7**	

Q	Solution	Marks	Total	Comments
5 (a)	$\bar{D}$ has a **normal** distribution with	B1		Normal
	mean = **0**	B1		CAO
	and variance = $\frac{\sigma^2}{n} + 1.5^2 \times \frac{\sigma^2}{n}$	M1		Must have (+ sign) & (1.5 or 1.5^2) but allow no ($\div n$)
	= $\frac{\mathbf{3.25\sigma^2}}{\boldsymbol{n}}$	A1	**4**	OE single expression
(b)	H_0: $\mu_{XL} = 1.5\mu_L$ H_1: $\mu_{XL} > 1.5\mu_L$	B1		B1 both; allow any valid notation
	1% $\Rightarrow$ z = **2.32 to 2.33**			AWFW (2.3263)
	or	B1		
	p-value of z-calculated = **0.004 to 0.005 < 0.01**			AWFW (0.004463)
	$z = \frac{2265 - (1.5 \times 1508)}{\sqrt{\frac{3.25 \times 4.5^2}{50}}} = \frac{3.0}{\sqrt{1.31625}}$	M1		Numerator; allow (2265 – 1508)
		M1		Denominator; allow $\sqrt{2 \times 4.5^2 / 50}$ OE
	= **2.61 to 2.62**	A1		AWFW (2.61488)
	Evidence, at 1% level, that **claim is supported**	Adep1	**6**	Dep on at least one M1 scored, z-value and CV
		Total	**10**	

Q	Solution	Marks	Total	Comments
6 (a)(i)	$V(X_1 + X_2) = V(X_1) + V(X_2) + 2\text{Cov}(X_1, X_2)$ Thus: $140^2 = 120^2 + 120^2 + 2\text{Cov}(X_1, X_2)$ Thus: $2\text{Cov}(X_1, X_2)$ = **19600 – 14400 – 14400 (= –9200)** $\Rightarrow \text{Cov}(X_1, X_2)$ = **–4600**	M1 A1 A1	 **3**	Used CAO AG
(ii)	$E(X_1 - X_2) = 1000 - 1000$ = **0** $V(X_1 - X_2) = 120^2 + 120^2 - (2 \times -4600)$ = **38000** **or** $\text{Sd}(X_1 - X_2)$ = **194 to 195** P(Difference > 250) = $P(\lvert X_1 - X_2 \rvert > 250) = 2 \times P\left(Z > \frac{250 - 0}{\sqrt{38000}}\right)$ = 2 × P(Z > 1.28) = 2 × [1 – (0.899 to 0.901)] = **0.2(00)**	B1 B1 M1 m1 A1	 **5**	CAO; may be implied CAO AWFW (194.936) Standardising 250 using c's mean & c's standard deviation Allow 'no 2 ×' Area change; allow 'no 2 ×' AWRT (0.19968)
(b)	$Y + B$ has: Mean = **2500** and Variance = $140^2 + 40^2$ = **21200** **or** Standard deviation = **145 to 146** P($Y + B$ < 2750) = $P\left(Z < \frac{2750 - 2500}{\sqrt{21200}}\right)$ = P(Z < **1.72**) = **0.957**	B1 B1 M1 A1 A1	 **5**	CAO CAO AWFW (145.602 Standardising 2750 using c's mean & c's standard deviation AWRT; ignore inequality and sign AWRT (0.95701)
		Total	**13**	

Q	Solution	Marks	Total	Comments
7 (a) (i)	$\underline{X \sim \text{Po}(\lambda)}$			
	$\text{E}(X) = \sum_{x=0}^{\infty} x \times \frac{\text{e}^{-\lambda}\lambda^x}{x!} =$	M1		Used; ignore limits until A1
	$\lambda\text{e}^{-\lambda} \times \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} =$	M1		Factor of at least λ Division of $x!$ by x
	$\lambda\text{e}^{-\lambda} \times \text{e}^{\lambda} = \underline{\underline{\lambda}}$	A1	3	AG; fully correct solution
(ii)	$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$			
	$= \text{E}[X(X-1)] + \text{E}(X) - [\text{E}(X)]^2$	M1		Used (Other derivations are possible)
	$= \lambda^2 + \lambda - \lambda^2 = \underline{\underline{\lambda}}$	A1	2	CAO
(b)(i)	$\text{P}(X = m) \geq \text{P}(X = m-1)$ and $\text{P}(X = m) \geq \text{P}(X = m+1) \Rightarrow$			
	$\frac{\text{e}^{-\lambda}\lambda^m}{m!} \geq \frac{\text{e}^{-\lambda}\lambda^{m-1}}{(m-1)!}$ and $\frac{\text{e}^{-\lambda}\lambda^m}{m!} \geq \frac{\text{e}^{-\lambda}\lambda^{m+1}}{(m+1)!}$	M1		Use of Po(λ) for $x = m$
		M1		Either inequality (accept = sign)
	$\underline{\underline{m \leq \lambda \quad \text{and} \quad m \geq \lambda - 1}}$	A1	3	AG; fully correct solution
(ii)	Given $\lambda = 4.9 \Rightarrow m = \underline{\underline{4}}$	B1		CAO
	$\text{P}(X = 4) = \frac{\text{e}^{-4.9}4.9^4}{4!} = \underline{\underline{0.178 \text{ to } 0.179}}$	B1	2	AWFW (0.178867)
(iii)	Given $\text{SD}(Y) = 15.5 \Rightarrow$ $\lambda = \text{Var(Y)} = \underline{\underline{15.5^2}} = \underline{\underline{240.25}}$	B1		Either CAO
	Mode, $d = \underline{\underline{240}}$	B1F		F on λ providing an integer value
	$\text{P}(Y_\text{P} \geq \text{d}) = \text{P}(Y_\text{N} > d - 0.5) =$	B1		Accept use of 'd'
	$\text{P}\left(Z > \frac{239.5 - 240.25}{15.5}\right) =$	M1		Standardising ($d - 0.5$, d or $d + 0.5$) with 15.5^2 and 15.5; do **not** accept use of 'd'
	$\text{P}(Z > -0.05) = \underline{\underline{0.515 \text{ to } 0.52}}$	A1	5	AWFW (0.5193)
		Total	**15**	