AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Pure Core 3

Wednesday 12 June 2019 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The curve with equation $y = x^3 + 3x - 8$ intersects the x-axis at the point A, where $x = \alpha$.

(a) Show that α lies between 1.4 and 1.6.

[2 marks]

(b) Show that the equation $x^3 + 3x - 8 = 0$ can be rearranged into the form

$$x = \sqrt[3]{(8 - 3x)}$$

[1 mark]

(c) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(8 - 3x_n)}$$

with $x_1 = 1.4$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(d) **Figure 1**, on the opposite page, shows parts of the graphs of $y = \sqrt[3]{(8 - 3x)}$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

[2 marks]

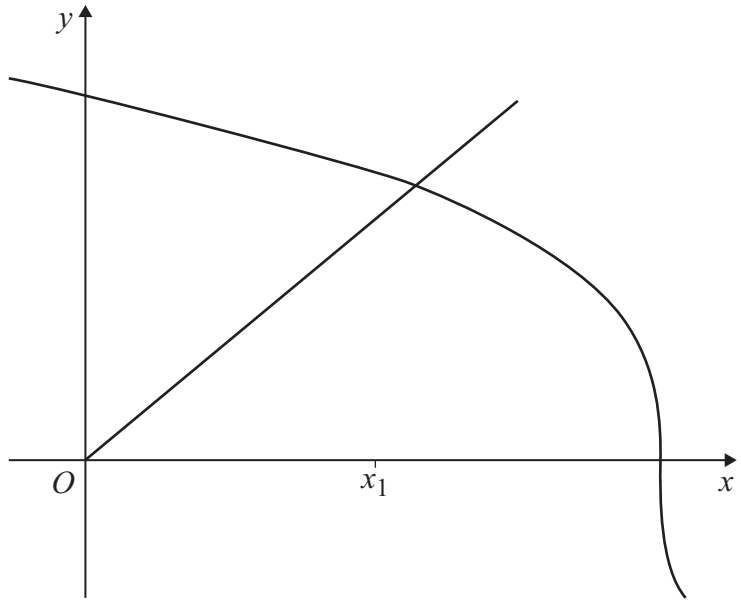
QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

Figure 1



2 The functions f and g are defined by

$$\mathrm{f}(x) = \sqrt{4x - 9}, \quad \text{for } x \geqslant 2.25$$

$$\mathrm{g}(x) = \frac{12}{x}, \quad \text{for all real values of } x,\ x \neq 0$$

(a) State the range of f.

[1 mark]

(b) (i) Find $\mathrm{fg}(x)$.

[1 mark]

(ii) Solve the equation $\mathrm{fg}(x) = 3$.

[2 marks]

(c) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$.

[3 marks]

(ii) Solve the equation $2\mathrm{f}^{-1}(x) = 29$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

3 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = \ln(3x + 4)$.

[4 marks]

(b) The **normal** to the curve $y = \ln(3x + 4)$ at the point $P(2,\ \ln 10)$ intersects the x-axis at the point A.

Find the exact value of the x-coordinate of A.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	**Answer space for question 3**

4 (a) (i) Find $\int \frac{x}{2x^2 + 1}\,\mathrm{d}x$.

[2 marks]

(ii) Hence find the exact value of $\int_{0.5}^{2.5} \frac{x}{2x^2 + 1}\,\mathrm{d}x$.

[2 marks]

(b) Use the mid-ordinate rule with five strips to find an estimate for $\int_{0.5}^{2.5} \ln\left(\frac{x}{2x^2 + 1}\right)\mathrm{d}x$, giving your answer to two decimal places.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

QUESTION PART REFERENCE	**Answer space for question 4**

5 (a) Solve the equation

$$\operatorname{cosec}(2x - 10°) = 1.5$$

giving all values of x to the nearest $0.1°$ in the interval $0° < x < 180°$.

[3 marks]

(b) Solve the equation

$$4\cot^2(2x - 10°) = 11 - 4\operatorname{cosec}(2x - 10°)$$

giving all values of x to the nearest $0.1°$ in the interval $0° < x < 180°$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE	**Answer space for question 5**

6 (a) Given that $e^{-0.5x} = 5$, find the exact value of x.

[2 marks]

(b) Use integration by parts to find $\int x e^{-0.5x} \, dx$.

[4 marks]

(c) The region bounded by the curve $y = e^{-0.5x} + 3x$, the x-axis and the lines $x = 0$ and $x = 1$ is rotated through 2π radians about the x-axis to form a solid.

Find the volume of the solid generated, giving your answer in an exact form.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

7 (a) Given that $y = \dfrac{2x - 5}{x^2 - 4}$, use the quotient rule to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax^2 + bx + c}{(x^2 - 4)^2}$$

where a, b and c are integers.

[3 marks]

(b) A curve has equation $y = \dfrac{2x - 5}{x^2 - 4}$.

(i) Find the values of the coordinates of the stationary points of the curve.

[3 marks]

(ii) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and hence determine the nature of these stationary points.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE	**Answer space for question 7**

QUESTION PART REFERENCE	**Answer space for question 7**

QUESTION PART REFERENCE	**Answer space for question 7**

8 For this question assume $0 < x < 0.5$

(a) Using the **product rule**, show that if $y = (\sin x)(\cos x)^{-1}$ then

$$\frac{dy}{dx} = \sec^2 x$$

[3 marks]

(b) Given that $x = \frac{1}{2}\sin u$ show that

$$\frac{\sqrt{(1-4x^2)}}{2x} = \cot u$$

[2 marks]

(c) Use the substitution $x = \frac{1}{2}\sin u$ to find

$$\int \frac{1}{(1-4x^2)^{\frac{3}{2}}}\,dx$$

giving your answer in terms of x.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE	**Answer space for question 8**

QUESTION PART REFERENCE	**Answer space for question 8**

QUESTION PART REFERENCE	**Answer space for question 8**

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED