A-level
MATHEMATICS
MPC3

Pure Core 3

Mark scheme

June 2019

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

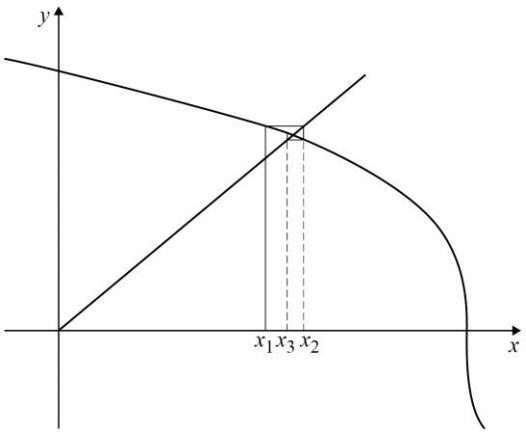
M	mark is for method
dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, the principal examiner may suggest that we award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$f(x) = x^3 + 3x - 8$ f(1.4) = –1.[1….] f(1.6) = 0.9[…] Change of sign(or different signs) $\Rightarrow 1.4 < \alpha < 1.6$	**M1** **A1**	**2**	Both values correct Must have both statement and interval in words or symbols
(b)	$x^3 + 3x - 8 = 0$ $x^3 = 8 - 3x$ $x = \sqrt[3]{8 - 3x}$	**B1**	**1**	Must see this line All correct and no errors seen
(c)	$x_2 = 1.560$ $x_3 = 1.492$	**B1** **B1**	**2**	
(d)		**M1** **A1**	**2**	Vertical line from x_1 to the curve, seen or implied, and then horizontal to $y = x$ All correct with 2nd vertical and horizontal lines (only required above the '$y = x$' line), and x_2, x_3 labelled on the x-axis
	Total		**7**	

Notes:

(a) Allow "x", "root" for α but not "it"

Q2	Solution	Mark	Total	Comment
(a)	$f(x) \geq 0$	**B1**	**1**	
(b)(i)	$fg(x) = \sqrt{\frac{48}{x} - 9}$	**B1**	**1**	
(ii)	$\sqrt{\frac{48}{x} - 9} = 3$ $\frac{48}{x} = 18$ $x = \frac{8}{3}$	**M1** **A1** **2**		
(c)(i)	$x = \sqrt{4y - 9}$ $x^2 + 9 = 4y$ $[f^{-1}(x)] = \frac{x^2 + 9}{4}$	**M1** **M1** **A1** **3**		Either order for **M1 M1** Interchange x and y Correctly eliminating square root. ACF
(c)(ii)	$\frac{x^2 + 9}{4} = 14.5$ $x^2 + 9 = 58$ $x = 7$ or $x = \pm 7$ Reject $x = -7$, PI Hence $x = 7$	**M1** **A1** **E1** **3**		
	Total		**10**	

Notes:
(a)
(c)(ii) Final answer of $x = \pm 7$ scores **M1A1E0**

Q3	Solution	Mark	Total	Comment
(a)	Stretch I Parallel to x-axis II SF 1/3 III then	**M1** **A1**		I and II or III (or line $y = 0$) I + II + III
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = -4/3$	**B1** **B1**		
	OR			
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = -4$ then Stretch I Parallel to x-axis II SF 1/3 III	**(B1)** **(B1)** **(M1)** **(A1)**		As above
	OR (either order)			
	Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = -4/3$ Translation $\begin{bmatrix} 0 \\ k \end{bmatrix}$ $k = \ln 3$	**(B1)** **(B1)** **(M1)** **(A1)**	**4**	
(b)	$\frac{dy}{dx} = \frac{3}{3x+4}$ oe	**M1**		
	At $x = 2$, Gradient $= 0.3$	**A1**		
	Equation of normal: $y - \ln 10 = -\frac{10}{3}(x-2)$	**M1**		Allow –1/(their grad)
	Intersects x – axis: $x = 0.3\ln 10 + 2$ **ACF**	**A1**	**4**	
	Total		**8**	

Q4	Solution	Mark	Total	Comment
(a)(i)	$[\int \quad =] \frac{1}{4}\ln(2x^2+1)+c$	M1 A1	2	$a\ln(2x^2+1)$ $a = 0.25$ and must have '+ c'
(a)(ii)	$\frac{1}{4}(\ln\frac{27}{2} - \ln\frac{3}{2})$	dM1		Correct subst of limits into $a\ln(2x^2+1)$
	$= \frac{1}{2}\ln 3$	A1	2	**OE**
(b)	<table><tr><td>x</td><td>y</td></tr><tr><td>0.7</td><td>–1.040</td></tr><tr><td>1.1</td><td>–1.134</td></tr><tr><td>1.5</td><td>–1.299</td></tr><tr><td>1.9</td><td>–1.465</td></tr><tr><td>2.3</td><td>–1.616</td></tr></table>	B1 B1		All 5 correct x values (and no extras used) PI by correct y values At least 4 correct y in exact form or decimal values, rounded or truncated to 3dp or better (in table or formula) (PI by correct answer)
	$0.4\times[-1.040-1.134-1.299-1.465-1.616]$	M1		Correct use of mid-ordinate rule using 0.4 and their 5 y values (of which 4 are correct), either listed or totalled.
	$= -2.62$	A1	4	CAO
	Total		**8**	

Q5	Solution	Mark	Total	Comment
(a)	$2x-10=41.8$ $[x=]$ 25.9 $[x=]$ 74.1	**M1** **A1** **A1**	**3**	PI by a correct final answer No other answers in range (ignore answers outside range)
(b)	$4\cot^2 Y = 11-4\operatorname{cosec} Y$ $4\operatorname{cosec}^2 Y = 15-4\operatorname{cosec} Y$ $4\operatorname{cosec}^2 Y+4\operatorname{cosec} Y-15=0$ $(2\operatorname{cosec} Y+5)(2\operatorname{cosec} Y-3)=0$ $\operatorname{cosec} Y = 1.5, -2.5$ $\operatorname{cosec} Y = -2.5$ –23.6 $[x=]$ 25.9, 74.1, 106.8, 173.2	**M1** **dM1** **A1** **B1** **B1**	**5**	Correct use of trig identity PI by a final answer of 106.8 or 173.2
	OR			
	$4\cos^2 Y = 11\sin^2 Y - 4\sin Y$ $15\sin^2 Y - 4\sin Y - 4 = 0$ $(5\sin Y+2)(3\sin Y-2)=0$ $\sin Y = -0.4, 0.667$	**(M1)** **(dM1)** **(A1)**		Correct use of trig identity
	Total		**8**	

Q6	Solution	Mark	Total	Comment
(a)	$-\frac{1}{2}x = \ln 5$ $x = -2\ln 5$	**M1** **A1**	**2**	ACF
(b)	$u = x \quad \frac{dv}{(dx)} = e^{-0.5x}$ $\frac{du}{(dx)} = 1 \quad v = -2e^{-0.5x}$ $\int = -2xe^{-0.5x} - \int -2e^{-0.5x}\ (dx)$ $\int = -2xe^{-0.5x} - 4e^{-0.5x}\ [+c]$ oe	**M1** **A1** **dM1** **A1**	**4**	All 4 terms in this form with $\frac{du}{dx}$ and $\int dx$ attempted All correct Correct substitution of their terms into the parts formula
(c)	$(V =)\pi\int_0^1 (e^{-0.5x} + 3x)^2 dx$ $[\int (e^{-0.5x} + 3x)^2 dx = \int] e^{-x} + 6xe^{-0.5x} + 9x^2\ [dx]$ $= -e^{-x} + 6(-2xe^{-0.5x} - 4e^{-0.5x}) + 3x^3$ $= [-e^{-1} + 6(-2e^{-0.5} - 4e^{-0.5}) + 3] - [-1 + 6 \times -4]$ $= 28 - e^{-1} - 36e^{-0.5}$ $V = \pi \times (28 - e^{-1} - 36e^{-0.5})$	**B1** **B1** **M1** **A1** **dM1** **A1**	**6**	Must include π, limits and dx Correct expansion Correct integration FT their part (b), middle term must be of correct form Correct subst of limits CAO
	Total		**12**	

Q7	Solution	Mark	Total	Comment
(a)	$[\frac{dy}{dx} =] \frac{(x^2-4)\times 2-(2x-5)\times 2x}{(x^2-4)^2}$	**M1**		$\frac{(x^2-4)\times k-(2x-5)\times lx}{(x^2-4)^2}$
		A1		
	$[= \frac{2x^2 - 8 - 4x^2 + 10x}{(x^2 - 4)^2}]$			
	$= \frac{-2x^2+10x-8}{(x^2-4)^2}$	**A1**	**3**	
(b)(i)	$\frac{-2x^2+10x-8}{(x^2-4)^2} = 0$			
	$x = 1,\ 4$	**M1**		Solving their quadratic equ
	$x = 1,\ y = 1$	**A1**		
	$x = 4,\ y = 0.25$	**A1**	**3**	
(b)(ii)	$\frac{d^2y}{dx^2} =$			
	$\frac{(x^2-4)^2(-4x+10)-(-2x^2+10x-8)(x^2-4)\times(4x)}{(x^2-4)^4}$	**M1**		Condone slips in $(-4x + 10)$ and $(4x)$ for **M** mark
		A1		
	$y' = 0,\ y'' = \frac{-4x+10}{(x^2-4)^2}$			
	$x = 1,\ y'' = \frac{2}{3} > 0$ Min pt	**M1**		Allow 6 / (+ve) > 0
	$x = 4,\ y'' = -\frac{1}{24} < 0$ Max pt	**A1**		Allow –6 / (+ve) < 0
			4	
	Total		**10**	

Q8	Solution	Mark	Total	Comment
(a)	$(\frac{dy}{dx})$			
	$=\sin x\times -1\times(\cos x)^{-2}\times -\sin x-\cos x(\cos x)^{-1}$	**M1**		Attempt at product rule
		A1		
	$=\frac{\sin^2 x}{\cos^2 x}+1$			
	$=\tan^2 x+1$			
	$=\sec^2 x$	**A1**		Must see 'a middle line' **AG**, all correct and no errors seen
			3	
(b)	$\frac{\sqrt{(1-4x^2)}}{2x}=\frac{\sqrt{1-\sin^2 u}}{\sin u}$	**M1**		
	$=\frac{\cos u}{\sin u}$			
	$=\cot u$	**A1**		Must see a middle line
			2	
(c)	$\frac{dx}{du}=0.5\cos u$ oe	**B1**		Correct expression for $\frac{du}{dx}$ or du or dx
	$\int\frac{1}{(1-4(0.5\sin u)^2)^{1.5}}\times 0.5\cos u[du]$	oe **M1**		Replacing all terms in x to all in terms of u, including replacing dx, but condone omission of du
		A1		All correct, must see du here or on later line
	$=0.5\int\frac{1}{(\cos^2 u)^{1.5}}\times\cos u[du]$	**dM1**		Correct use of trig identity
	$=0.5\int\sec^2 u[du]$	**dM1**		Correct simplification of powers
	$=0.5\tan u$	**A1**		Using part (a)
	$[\tan u=\frac{2x}{\sqrt{1-4x^2}}]$			
	$\int\ =\frac{x}{\sqrt{1-4x^2}}$	**A1**		
			7	
	Total		**12**	