AQA

Please write clearly in block capitals.

Centre number						Candidate number				

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________

A-level
MATHEMATICS
Unit Pure Core 3

Wednesday 13 June 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Find $\dfrac{dy}{dx}$ when $y = (5 + 3x^2)^{\frac{1}{2}}$.

[2 marks]

(b) Find $\displaystyle\int (1 + \sin 4x)\, dx$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

2 (a) By writing $\cot x$ as $\dfrac{\cos x}{\sin x}$, use the quotient rule to show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\operatorname{cosec}^2 x$.

[2 marks]

(b) The curve with equation $x = \dfrac{\pi}{12} - y + \cot 3y$ is defined for $0 < y < \dfrac{\pi}{3}$.

(i) Find $\dfrac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

[2 marks]

(ii) Hence find the exact equation of the tangent to the curve at the point $\left(1, \dfrac{\pi}{12}\right)$, giving your answer in the form $y = mx + c$, where m is a rational number.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	Answer space for question 2

3 (a) (i) Sketch the graph of $y = \ln(2x)$, stating the coordinates of any point where the curve crosses the coordinate axes.

[2 marks]

(ii) Describe a sequence of two geometrical transformations that maps the graph of $y = \ln(2x)$ onto the graph of $y = \ln(3 + 4x)$.

[4 marks]

(b) (i) Use Simpson's rule with five ordinates (four strips) to find an approximate value for $\int_1^3 \ln(3 + 4x)\,\mathrm{d}x$, giving your answer to five significant figures.

[4 marks]

(ii) Given that the value of $\int_1^3 \ln(3 + 4x)\,\mathrm{d}x$ is A,

show that the value of $\int_1^3 \ln(3\mathrm{e} + 4\mathrm{e}x)\,\mathrm{d}x$ is $A + 2$.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE

Answer space for question 3

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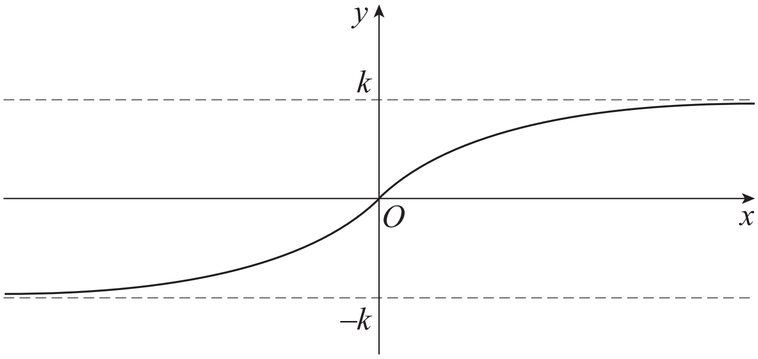
QUESTION PART REFERENCE	Answer space for question 3

4 (a) The function f, given by $\mathrm{f}(x) = \tan^{-1} x$, is defined for all values of x.
The graph of $y = \mathrm{f}(x)$ is sketched below.



(i) The range of f is $-k < \mathrm{f}(x) < k$. State the exact value of k.

[1 mark]

(ii) On **Figure 1** below, sketch the graph of $y = |\tan^{-1} x|$.

[1 mark]

(iii) By drawing a suitable straight line on your sketch on **Figure 1**, show that the equation

$$|\tan^{-1} x| - 7x - 3 = 0$$

has exactly one real root.

[2 marks]

(b) The real root of the equation $|\tan^{-1} x| - 7x - 3 = 0$ is α. Show that α lies between -0.4 and -0.3.

[2 marks]

(c) Use the iterative formula

$$x_{n+1} = \frac{1}{7}(|\tan^{-1} x_n| - 3) \text{ with } x_1 = -0.4$$

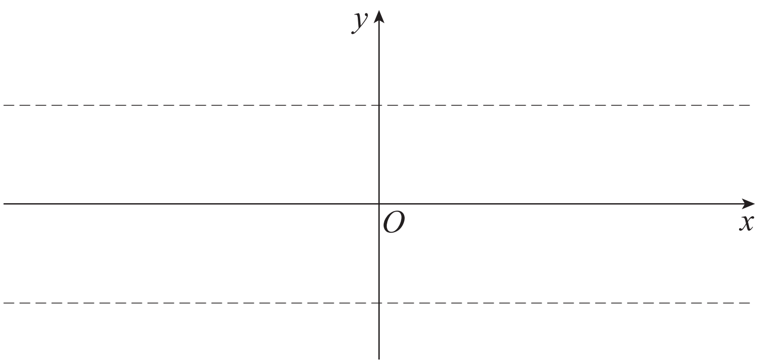
to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

QUESTION PART REFERENCE | **Answer space for question 4**

(a)(i) $k =$ ....

Figure 1



QUESTION PART REFERENCE	Answer space for question 4

5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \sqrt{2x+3} \qquad \text{for } x \geqslant -1$$

$$\mathrm{g}(x) = x^2 + 4x \qquad \text{for all real values of } x$$

(a) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$.

[3 marks]

(ii) State the domain of f^{-1}.

[1 mark]

(b) Find the range of g.

[3 marks]

(c) (i) Find $\mathrm{gf}(x)$.

[1 mark]

(ii) Solve the equation $\mathrm{gf}(x) = 21$.

[5 marks]

QUESTION PART REFERENCE	Answer space for question 5

QUESTION PART REFERENCE	Answer space for question 5

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QUESTION PART REFERENCE	Answer space for question 5

Turn over ▶

6 Use the substitution $u = 2 + \ln x$ to show that

$$\int_1^{e} \frac{\ln x}{x(2+\ln x)^2}\,dx = p + \ln q$$

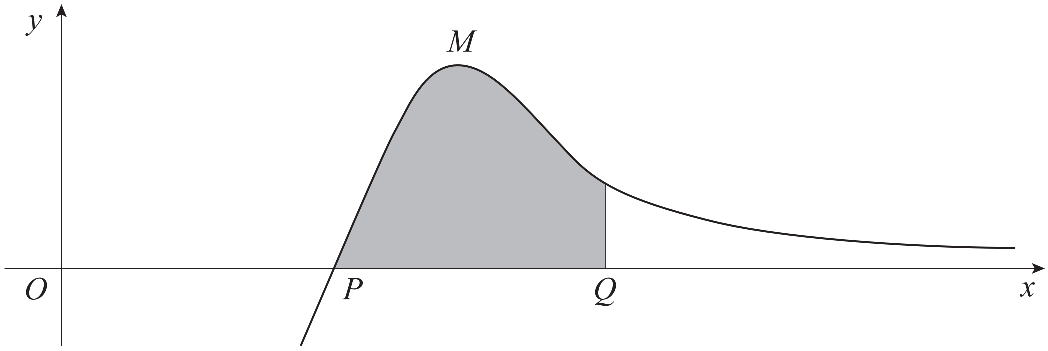
where p and q are rational numbers.

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	Answer space for question 6

7 Part of a curve is sketched below.



The curve has equation $y = (x - 1)\,e^{-3x}$.

(a) The curve has a stationary point M. Show that the x-coordinate of M is $\frac{4}{3}$.

[3 marks]

(b) (i) By using integration by parts twice, find

$$\int (x-1)^2 e^{-6x}\,dx$$

[6 marks]

(ii) The curve crosses the x-axis at the point P. The point Q has coordinates $(2, 0)$. The shaded region bounded by the curve, the x-axis from P to Q and the line $x = 2$ is rotated through 2π radians about the x-axis. Find the exact value of the volume of the solid generated.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

QUESTION PART REFERENCE	Answer space for question 7

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8 (a) Show that the expression $\dfrac{\sec\theta}{\sec\theta - 1} + \dfrac{\sec\theta}{\sec\theta + 1}$ can be written as $2\,\text{cosec}^2\,\theta$.

[3 marks]

(b) Hence solve the equation

$$\frac{\sec(2x+0.4)}{\sec(2x+0.4)-1} + \frac{\sec(2x+0.4)}{\sec(2x+0.4)+1} = 8 - \cot(2x+0.4)$$

giving your answers in radians to three significant figures in the interval $0 < x < \pi$.

[7 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE	Answer space for question 8

QUESTION PART REFERENCE	Answer space for question 8

END OF QUESTIONS