A-LEVEL

Mathematics

MPC3 – Pure Core 3
Mark scheme

6360
June 2016

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\left[\frac{\mathrm{dy}}{\mathrm{dx}}=\right] m(4x+1)^3\cos 2x+n(4x+1)^2\sin 2x$	**M1**		$m,n\neq 0$
	$m=2$ and $n=4\times 3[=12]$ isw	**A1**	**2**	
(b)	$\left[\frac{\mathrm{dy}}{\mathrm{dx}}=\right]\frac{(3x^2+4)4x-(2x^2+3)6x}{(3x^2+4)^2}$ oe	**M1**		Or $(2x^2+3)(-1)(3x^2+4)^{-2}6x+(3x^2+4)^{-1}4x$
	$=\frac{-2x}{(3x^2+4)^2}$	**A1**	**2**	
(c)	$\left[\frac{\mathrm{dy}}{\mathrm{dx}}=\right]\frac{1}{\frac{2x^2+3}{3x^2+4}}\times\text{their } b(i)$ PI	**M1**		'their b(i)' must be in the correct form $\frac{kx}{(3x^2+4)^2}$
	$\left[\frac{\mathrm{dy}}{\mathrm{dx}}=\right]\frac{(3x^2+4)}{(2x^2+3)}\times\frac{-2x}{(3x^2+4)^2}$ isw	**A1**		**Or** (using rules of logs) $y=\ln(2x^2+3)-\ln(3x^2+4)$ $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{ax}{2x^2+3}-\frac{bx}{3x^2+4}$ M1 $a>0,b>0$ $a=4,\ \ b=6$ A1
	$\left(=\frac{-2x}{(2x^2+3)(3x^2+4)}\right)$		**2**	
	Total		**6**	

Notes: Allow recovery from poor use of brackets in each part.

(a) If expanded, $\frac{\mathrm{dy}}{\mathrm{dx}}=(ax^2+bx+c)\sin 2x+(dx^3+ex^2+fx+1)2\cos 2x$ M1

$a=192,\ b=96,\ c=12,\ d=64,\ e=48,\ f=12$ A1

(b) For M1, $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\pm(3x^2+4)4x\pm(2x^2+3)6x}{(3x^2+4)^2}$ or $\pm(2x^2+3)(-1)(3x^2+4)^{-2}6x\pm(3x^2+4)^{-1}4x$

For A1, accept p= – 2

<table>
<tr><th>Q2</th><th>Solution</th><th>Mark</th><th>Total</th><th>Comment</th></tr>
<tr><td>a</td><td>$\text{f}(x)=x^x-5$ PI</td><td></td><td></td><td>(or reverse)</td></tr>
<tr><td></td><td>f(2) = – 1</td><td></td><td></td><td></td></tr>
<tr><td></td><td>f(3) = 22</td><td>M1</td><td></td><td>Both values correct</td></tr>
<tr><td></td><td>Change of sign(or different signs)
$\Rightarrow 2 < \alpha < 3$</td><td>A1</td><td></td><td>Must have both statement and interval in words or symbols
OR comparing 2 sides:
at 2, $2^2 < 5$;
at 3, $3^3 > 5$ (M1)</td></tr>
<tr><td></td><td></td><td></td><td>2</td><td>$\Rightarrow 2 < \alpha < 3$ (A1)</td></tr>
<tr><td>b</td><td>$(x^x = 5 \quad \Rightarrow \quad \ln x^x = \ln 5)$</td><td></td><td></td><td></td></tr>
<tr><td></td><td>$x \ln x = \ln 5$</td><td>M1</td><td></td><td>Taking logs and using rule of logs</td></tr>
<tr><td></td><td>$\ln x = \dfrac{\ln 5}{x}$</td><td>A1</td><td></td><td>Must see this line</td></tr>
<tr><td></td><td>$x = \text{e}^{\frac{\ln 5}{x}}$</td><td>A1</td><td>3</td><td>AG, all correct (including middle line)</td></tr>
<tr><td>c</td><td>$[x_2 =]2.236$</td><td>B1</td><td></td><td></td></tr>
<tr><td></td><td>$[x_3 =]2.054$</td><td>B1</td><td>2</td><td>Ignore any further values</td></tr>
<tr><td>di</td><td>
<table>
<tr><th>x</th><th>y</th></tr>
<tr><td>0.5</td><td>4.29289</td></tr>
<tr><td>0.7</td><td>4.22094</td></tr>
<tr><td>0.9</td><td>4.09047</td></tr>
<tr><td>1.1</td><td>3.88947</td></tr>
<tr><td>1.3</td><td>3.59354</td></tr>
<tr><td>1.5</td><td>3.16288</td></tr>
<tr><td>1.7</td><td>2.53531</td></tr>
</table>
</td><td>B1
B1</td><td></td><td>All 7 correct x values (and no extras used) PI by correct y values
At least 5 correct y in exact form or decimal values, rounded or truncated to 3dp or better (in table or formula) (PI by correct answer)</td></tr>
<tr><td></td><td>$\frac{1}{3}\times 0.2[4.2929+2.5353+4(4.2209$
$+3.8895+3.1629)+2(4.0905+3.5935)]$</td><td>M1</td><td></td><td>Correct use of Simpson’s rule using 1/3 and 0.2 oe and their 7 y values (of which 5 are correct to 2dp), either listed or totalled.</td></tr>
<tr><td></td><td>$=4.49$</td><td>A1</td><td>4</td><td>CAO</td></tr>
<tr><td>dii</td><td>6 - their (di)
= 1.51</td><td>M1
A1F</td><td>2</td><td>PI by correct answer
SC1 for – 1.51</td></tr>
<tr><td></td><td>Total</td><td></td><td>13</td><td></td></tr>
</table>

Notes:

a condone $2 \le x \le 3$, allow ‘x’, ’ root’ for α, but not ‘it’

di 4.49 with no working scores 0/4

dii 1.51 scores 2/2, 1.51… with NMS scores 0/2

Q3	Solution	Mark	Total	Comment
	$x^2 - 5x + 6[=0]$			
	$[x=]2,\ 3$ PI	**B1**		**B1** can be earned for any correct 2 solutions
	$x^2 + 5x - 6 = 0$			
	$[x=] -6,\ 1$ PI	**B1**		
				or
	$x \le -6$	**B1**		$-6 \ge x$
	$x \ge 3$	**B1**		$3 \le x$
	$1 \le x \le 2$	**B1**		And no extras seen
			5	
	Total		**5**	

Notes:
Correct inequalities implies correct critical values if not seen explicitly

A candidate may use a quartic to find the critical values, but marks are only earned for correct solutions as above, eg solutions of 1, 2 scores **B1**
If strict inequalities are used **throughout** then penalise 1 mark – but if some correct answers and some strict inequalities then mark as scheme.

Q4	Solution	Mark	Total	Comment
a	Stretch I [Parallel to] x[-axis] II (or line $y = 0$) [SF] 0.5 III	**M1** **A1**		I and II or III I + II + III
	then Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**M1**		**or** (2nd) Stretch [parallel to] y[-axis]
	$\begin{bmatrix} 2.5 \\ 0 \end{bmatrix}$	**A1**		SF e^{-5} (for the '2 stretch' method, if the 'y' direction stretch is first, marks **can only** be earned if there is a **second** stretch in 'x' direction. The stretches can be in either order)
	OR Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**(M1)**		
	$\begin{bmatrix} 5 \\ 0 \end{bmatrix}$	**(A1)**		
	then Stretch I [Parallel to] x[-axis] II [SF] 0.5 III	**(M1)** **(A1)**	**4**	I and II or III I + II + III
b	$\frac{dy}{dx} = 2e^{2x-5}$	**B1**		
	Grad normal = $-\frac{1}{\text{their gradient}}$	**B1F**		Condone expression in terms of x
	(equation normal) $y - e^{-1} = -\frac{e}{2}(x-2)$ oe	**B1**		Must be exact values
	(At A $y = 0$) $x = 2 + \frac{2}{e^2}$ oe	**M1**		Attempt to find at least one intercept from 'their' normal, subst $x = 0$ **or** $y = 0$ in any straight line equation
	(At B $x = 0$) $y = e + \frac{1}{e} = \frac{e^2+1}{e}$ oe	**A1**		Both x **and** y values correct
	$\left(\text{(Area =)}\ 0.5 \times \frac{(e^2+1)}{e} \times \frac{2(1+e^2)}{e^2}\right)$ $= \frac{(e^2+1)^2}{e^3}$	**A1**	**6**	
	Total		**10**	

Notes: (a) translation (accept translate, transla…), **must** be with a column vector to score M1
For '[parallel to] x[-axis]', DO NOT allow '$x = 0$'

Q5	Solution	Mark	Total	Comment
a	$[\mathrm{f}'(x)] = 16 - 2e^{2x}$	**B1**		
	$(\mathrm{f}'(x) = 0)$ $16 - 2\mathrm{e}^{2x} = 0$	**M1**		For equating their derivative to zero (must be of form $a + be^{2x}$)
	$x = \frac{1}{2}\ln 8$ oe	**A1**		Allow AWRT 1.04
	$[\mathrm{f}(x) =] 8\ln 8 - 8$ oe	**m1**		Correct subst of their x into f(x), Allow AWRT 8.63 or 8.64
	$\mathrm{f}(x) \leq 8\ln 8 - 8$ oe	**A1**		Must have exact form and correct notation, no **ISW**
			5	
b	$\mathrm{g}(x) = \frac{1}{x}$ oe	**M1**		
	$\mathrm{gg}(x) = x$	**A1**		**NMS** 2/2
			2	
	Total		**7**	

Notes:

(a) Allow **equivalent exact forms** for $8\ln 8 - 8$, but not decimal equivalent for final **A** mark
Must have simplified $e^{\ln 8}$

Q6	Solution	Mark	Total	Comment
a	$u=\ln 3x \qquad \frac{\mathrm{d}u}{(\mathrm{d}x)}=\frac{1}{x}$ oe	**B1**		PI by further work
	$\frac{\mathrm{d}v}{(\mathrm{d}x)}=\frac{1}{x^2} \qquad v=-x^{-1}$	**B1**		PI by further work
	$\int=-\frac{1}{x}\ln 3x-\int -x^{-1}\times\frac{1}{x}\ (\mathrm{d}x)$ oe	**M1**		Correct substitution of **their terms** into the parts formula
	$=-\frac{1}{x}\ln 3x-\frac{1}{x}\ (+c)$ oe	**A1**	**4**	
b	$(\mathrm{V}=)\pi\int_{\frac{1}{3}}^{1}(\frac{\ln 3x}{x})^2\mathrm{d}x$	**B1**		Must include π, (not 2 π), limits and dx (each seen at some stage, in this part)
	$[u=(\ln 3x)^2] \qquad \frac{\mathrm{d}u}{(\mathrm{d}x)}=2\ln 3x\times\frac{1}{x}$	**M1** **A1**		$\frac{\mathrm{d}u}{(\mathrm{d}x)}=k\ln 3x\times\frac{1}{x}$ $k=2$
	$\frac{\mathrm{d}v}{(\mathrm{d}x)}=x^{-2} \qquad v=-x^{-1}$			
	$\int(\frac{\ln 3x}{x})^2\mathrm{d}x=$ $-\frac{1}{x}(\ln 3\mathrm{x})^2-\int -x^{-1}\times 2\ln 3x\times\frac{1}{x}\ (\mathrm{d}x)$	**M1**		Correct substitution of **their terms** into the parts formula
	$[=-\frac{1}{x}(\ln 3x)^2+\int 2\frac{\ln 3x}{x^2}\mathrm{d}x]$			
	$=-\frac{1}{x}(\ln 3x)^2-\frac{2}{x}\ln 3x-\frac{2}{x}$	**A1**		
	$=[-(\ln 3)^2-2\ln 3-2]-[-3(\ln 1)^2-6\ln 1-6]$ (ln1 terms may be omitted)	**M1**		Correct subst into expression of the form $\frac{k}{x}(\ln 3x)^2+\frac{l}{x}\ln 3x-\frac{m}{x}$ and F(1)-F(1/3)
	$[V=]\ \pi(4\text{-}(\ln 3)^2-2\ln 3)$	**A1**		

	OR $u=\ln 3x \qquad \frac{\mathrm{d}v}{(\mathrm{d}x)}=\frac{\ln 3x}{x^2}$	**(M1)**		‘splitting’ in this way
	$\frac{\mathrm{d}u}{(\mathrm{d}x)}=\frac{1}{x}$ oe $\qquad v=-\frac{1}{x}\ln 3x-\frac{1}{x}$	**(A1)**		
	$\int =$ $\ln 3x\left(-\frac{1}{x}\ln 3x-\frac{1}{x}\right)-\int\left(-\frac{1}{x}\ln 3x-\frac{1}{x}\right)\frac{1}{x}(\mathrm{d}x)$	**(M1)**		Correct substitution of their terms into the parts formula
	$=-\frac{1}{x}(\ln 3x)^2-\frac{2}{x}\ln 3x-\frac{2}{x}$	**(A1)**		
			7	First **B1** and final 2 marks are as first method
	Total		**11**	

Notes

In both parts, the method mark for use of parts formula is earned for correct subst of **their** terms into the parts formula, with no restriction on **their** terms

(b) Condone $(\ln 3x)^2$ as $\ln^2 3x$, throughout

Q7	Solution	Mark	Total	Comment
(a)	$(\frac{\mathrm{dy}}{\mathrm{dx}})=-1\times(\cos x)^{-2}\times-\sin x$	**M1**		
	$=\frac{\sin x}{\cos^2 x}$ oe			
	$=\frac{\sin x}{\cos x}\times\frac{1}{\cos x}$ oe			Must see 'a middle line'
	$=\tan x\sec x$	**A1**		AG, all correct and no errors seen
			2	
(b)	$(\frac{\mathrm{dy}}{\mathrm{dx}}=)\ 2\sec^2 x-3\sec x\tan x$	**M1**		$m\sec^2 x+n\sec x\tan x$
	$(\frac{\mathrm{dy}}{dx}=0)$			
	$[\sec x](2\sec x-3\tan x)[=0]$ oe	**m1**		$[\sec x](m\sec x+n\tan x)[=0]$
	$\sin x=\frac{2}{3}$	**A1**		Finding **any** correct exact trig ratio
	$\cos x=\frac{\sqrt{5}}{3}$ $\tan x=\frac{2}{\sqrt{5}}$ $\sec x=\frac{3}{\sqrt{5}}$	**A1**		Finding a second correct exact trig ratio
	$y=2\times\frac{2}{\sqrt{5}}-3\times\frac{3}{\sqrt{5}}$	**M1**		For subst their exact values correctly into 'y' (PI by correct final answer following previous 4 marks earned)
	$y=-\sqrt{5}$	**A1** **CSO**	**6**	Must have used correct exact values throughout If second M mark is not earned, then **SC1** for AWRT –2.24 or $-\sqrt{5}$
	Total		**8**	

Notes: (a) For **M1**, condone 'dropping one minus sign' and/or poor use of brackets

Candidates must use chain rule to qualify for **M1**. Clear use of quotient rule scores 0/2

(c) If different approach then **m1** only earned when $a\sin x=b$ oe or $a\sec x=b$ oe is seen

Candidates could use trig identity to find tan x first, and then sec x

For ‘second value’, any of the two trig ratios, but not just sec *x* and cos *x*.
If second M mark is not earned, then **SC1,** eg a candidate could score M1m1A1A0M0 SC1

Q8	Solution	Mark	Total	Comment
	$\frac{\mathrm{d}u}{\mathrm{d}x} = 4$ oe	**B1**		Correct expression for $\frac{\mathrm{d}u}{\mathrm{d}x}$ or d*u* or d*x*
	$[u = 4x - 1]$ $4x = u + 1$ oe	**B1**		Correct term in *kx*, where *k* = 1, 2, 4
	$\int \frac{9-u}{2} \times \sqrt[3]{u} \times \frac{(\mathrm{d}u)}{4}$ oe	**M1**		Replacing all terms in *x* to all in terms of u, including replacing d*x*, but condone omission of d*u*
		A1		All correct, must see d*u* here or on next line
	$\left(= \frac{1}{8}\int 9u^{\frac{1}{3}} - u^{\frac{4}{3}}(\mathrm{d}u)\right)$ oe			
	$= \frac{1}{8}\left(\frac{3}{4} \times 9u^{\frac{4}{3}} - \frac{3}{7}u^{\frac{7}{3}}\right)$ oe	**m1**		Correct integration from an expression of the form $au^{\frac{1}{3}} + bu^{\frac{4}{3}}$ or $au^{\frac{1}{3}} + bu^{\frac{4}{3}} + cu^{\frac{1}{3}}$
	Limits $[x]_{0.25}^{0.5} = [u]_0^1$ may be seen earlier	**B1**		Or, correctly changing variable back into *x*
	$\left(= \frac{1}{8}\left[\left(\frac{27}{4} - \frac{3}{7}\right) - 0\right]\right)$			
	$= \frac{177}{224}$ oe	**A1**		allow equivalent fraction
	Total		**7**	
Notes: Candidates might not collect terms, but proceed as follows				

$\int \frac{5-2(\frac{u+1}{4})}{4} \times \sqrt[3]{u} \times (\mathrm{d}u)$	M1 A1
$\left(= \int \frac{5}{4}u^{\frac{1}{3}} - \frac{u^{\frac{4}{3}} + u^{\frac{1}{3}}}{8} (\mathrm{d}u) \right)$	
$= \frac{15}{16}u^{\frac{4}{3}} - \frac{3u^{\frac{7}{3}}}{56} - \frac{3u^{\frac{4}{3}}}{32}$	m1 etc

Q9	Solution	Mark	Total	Comment
ai	$(\sec^2 x - \tan^2 x = 1)$			
	$(\sec x + \tan x)(\sec x - \tan x) = \sec^2 x - \tan^2 x$	**M1**		Or correct **use of** $\sec^2 x = 1 + \tan^2 x$ in a correct expression
	$-5(\sec x + \tan x) = 1$ $\sec x + \tan x = -0.2$	**A1**	**2**	AG: no errors seen
ii	$2\sec x = -5.2$ or $2\tan x = 4.8$	**M1**		Correctly reducing to a linear equation in one trig function
	$\sec x = -2.6$	**A1**		PI by correct value for $\cos x$
	$\cos x = -\frac{5}{13}$ oe	**A1**	**3**	$\frac{a}{b}$ where a, b are correct integers
b	$\sec y = -2.6$			
	$[y =]\ [\pm]112.6^\circ$	**B1**		**AWRT** $[\pm]112.6^\circ$ PI by a correct final answer
	$2x - 70 = [\pm]$*their* y	**M1**		
	$x =$ -21.3°, [-88.7°],	**A1**	**3**	And no other extras in interval (ignore answers outside interval)
	Total		**8**	

Notes:

ai Alternative I,

$\left(\sin x = -\frac{12}{13}\right)$ correctly using $\sin^2 x + \cos^2 x = 1$ **M1**

($\cos x = -\frac{5}{13}$ and/or $\tan x = \frac{12}{5}$ **)**

leading to

$\sec x + \tan x = -0.2$ **A1** **AG**: no errors seen, must see a middle line

Also, the candidate could earn the M1A1 for part (ii) here (but only if part (ii) attempted, but final A1 mark must appear in part (ii)

ii Correct answer with no working scores 3/3, If **M0** scored, **SC2** for $\cos x = \mp\frac{5}{13}$ or $\frac{5}{13}$

b final answer **must** be to 1dp
If M0 scored, then SC1 for –21.3