

Centre Number						Candidate Number				
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Other Names										
Candidate Signature										



General Certificate of Education
Advanced Level Examination
June 2013

Mathematics

MPC3

Unit Pure Core 3

Thursday 6 June 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

- Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
 - Fill in the boxes at the top of this page.
 - Answer **all** questions.
 - Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
 - You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
 - Do not write outside the box around each page.
 - Show all necessary working; otherwise marks for method may be lost.
 - Do all rough work in this book. Cross through any work that you do not want to be marked.

- Information**
- The marks for questions are shown in brackets.
 - The maximum mark for this paper is 75.

- Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
 - You do not necessarily need to use all the space provided.

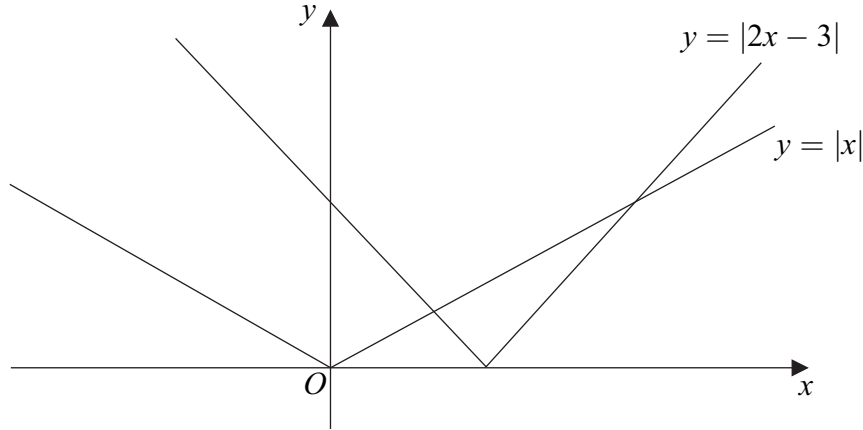
For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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9	
10	
TOTAL	



Answer **all** questions.

Answer each question in the space provided for that question.

1 The diagram below shows the graphs of $y = |2x - 3|$ and $y = |x|$.



(a) Find the x -coordinates of the points of intersection of the graphs of $y = |2x - 3|$ and $y = |x|$. (3 marks)

(b) Hence, or otherwise, solve the inequality

$$|2x - 3| \geq |x| \quad (2 \text{ marks})$$

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QUESTION
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Answer space for question 1

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2 (a) Given that $y = x^4 \tan 2x$, find $\frac{dy}{dx}$. (3 marks)

(b) Find the gradient of the curve with equation $y = \frac{x^2}{x-1}$ at the point where $x = 3$. (3 marks)

QUESTION
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Answer space for question 2



QUESTION
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Answer space for question 2

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3 (a) The equation $e^{-x} - 2 + \sqrt{x} = 0$ has a single root, α .

Show that α lies between 3 and 4. (2 marks)

(b) Use the recurrence relation $x_{n+1} = (2 - e^{-x_n})^2$, with $x_1 = 3.5$, to find x_2 and x_3 , giving your answers to three decimal places. (2 marks)

(c) The diagram on the opposite page shows parts of the graphs of $y = (2 - e^{-x})^2$ and $y = x$, and a position of x_1 .

On the diagram, draw a staircase or cobweb diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x -axis. (2 marks)

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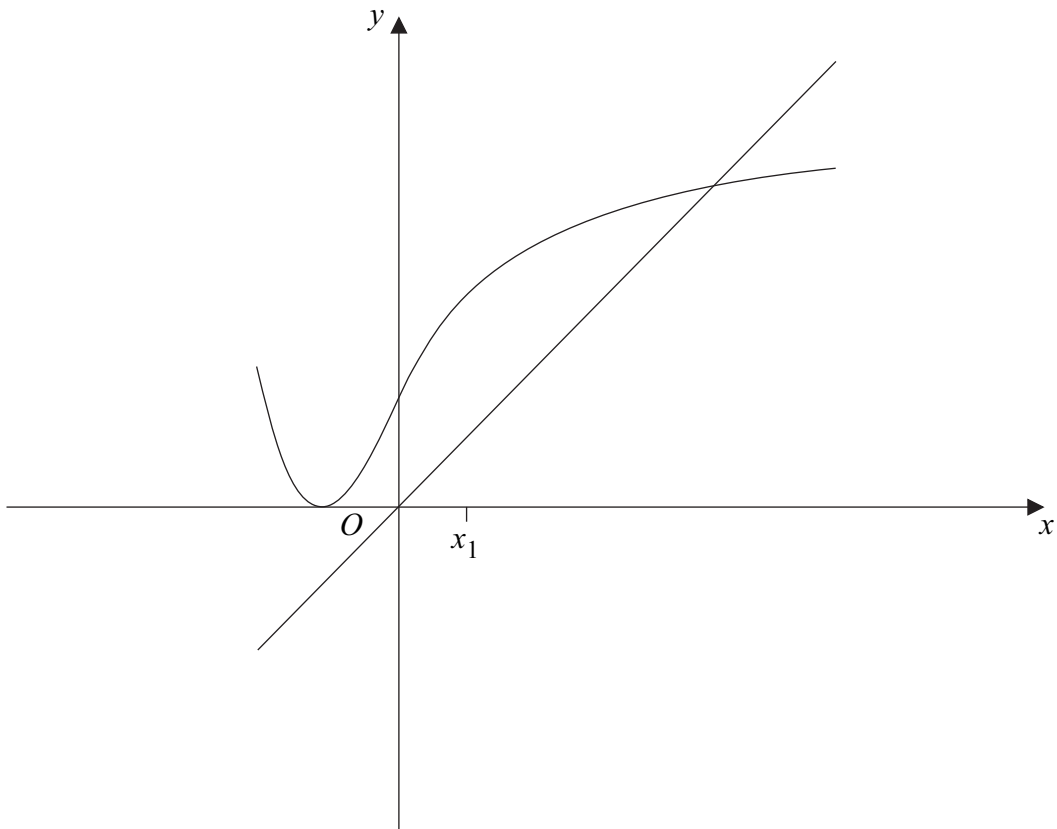


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Answer space for question 3

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(c)



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4 By forming and solving a quadratic equation, solve the equation

$$8 \sec x - 2 \sec^2 x = \tan^2 x - 2$$

in the interval $0 < x < 2\pi$, giving the values of x in radians to three significant figures. (7 marks)

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Answer space for question 4



QUESTION
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Answer space for question 4

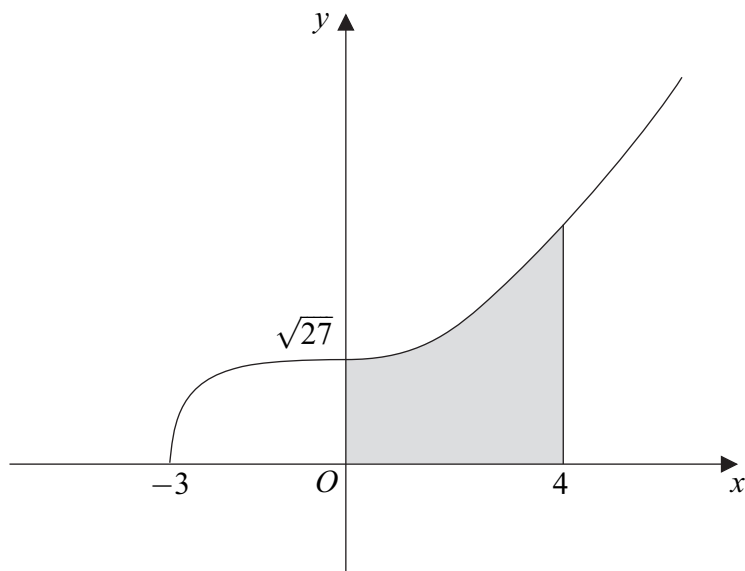
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5

The diagram shows a sketch of the graph of $y = \sqrt{27 + x^3}$.



- (a) The area of the shaded region, bounded by the curve, the x -axis and the lines $x = 0$ and $x = 4$, is given by $\int_0^4 \sqrt{27 + x^3} dx$.

Use the mid-ordinate rule with **five** strips to find an estimate for this area. Give your answer to three significant figures. (4 marks)

- (b) With the aid of a diagram, explain whether the mid-ordinate rule applied in part (a) gives an estimate which is smaller than or greater than the area of the shaded region. (2 marks)

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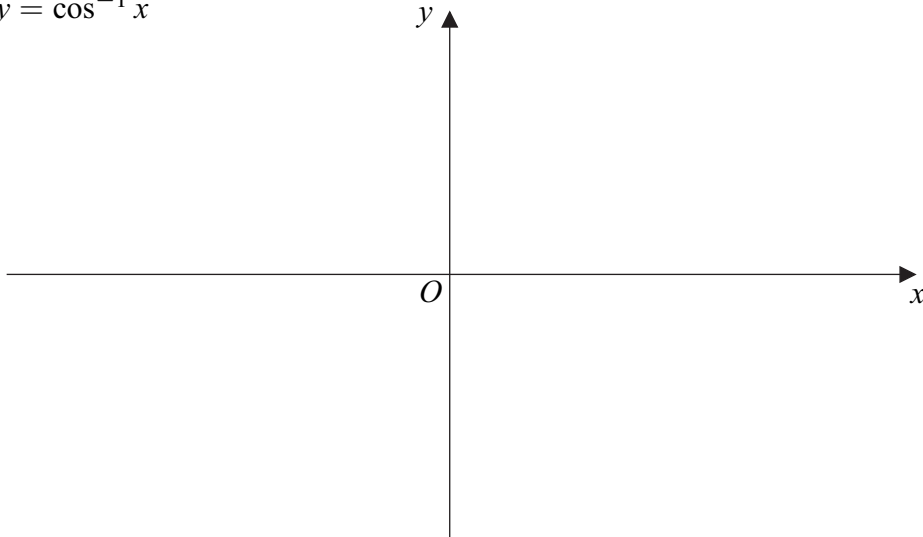


- 6 (a)** Sketch the graph of $y = \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. (2 marks)
- (b)** Sketch the graph of $y = \pi - \cos^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. (2 marks)

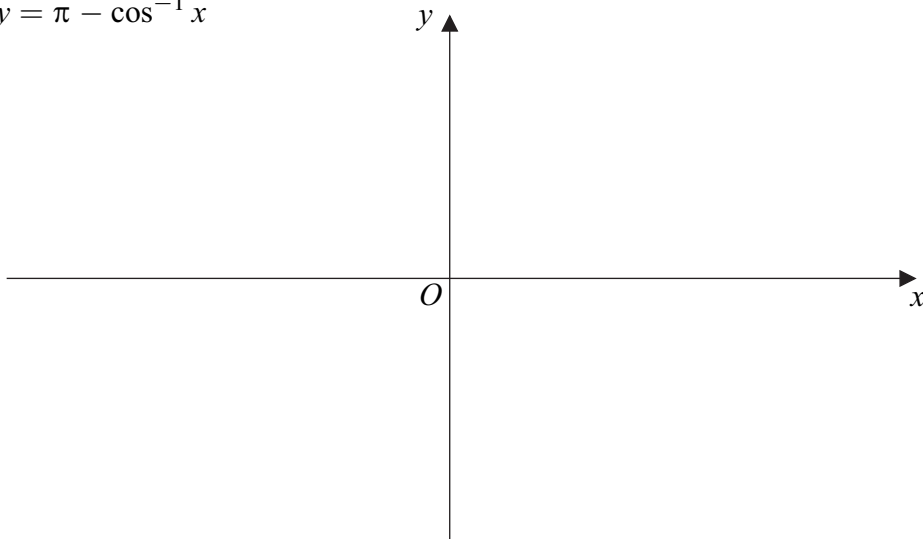
 QUESTION
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Answer space for question 6
(a)

$$y = \cos^{-1} x$$


(b)

$$y = \pi - \cos^{-1} x$$



QUESTION
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Answer space for question 6

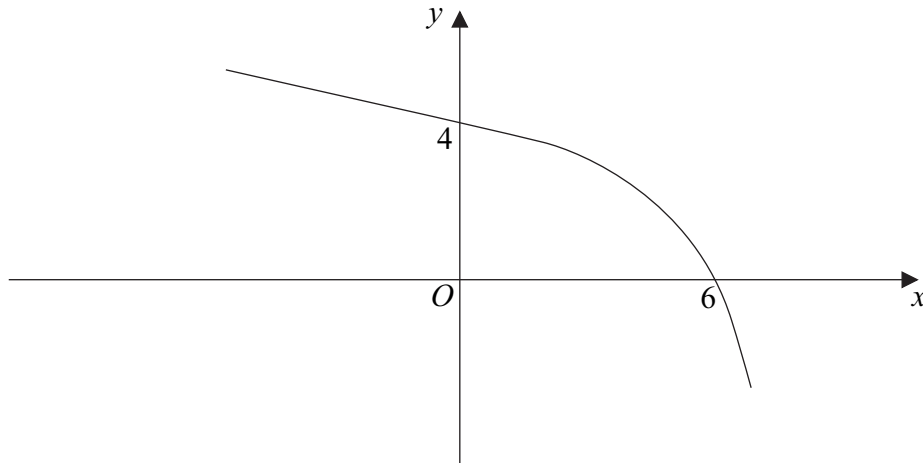
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The diagram shows a sketch of the curve with equation $y = f(x)$.



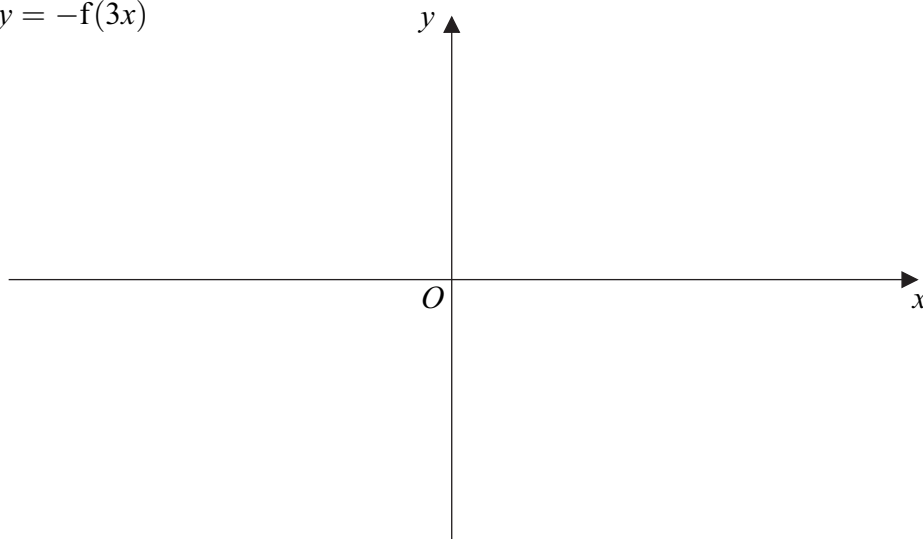
- (a) On **Figure 1**, below, sketch the curve with equation $y = -f(3x)$, indicating the values where the curve cuts the coordinate axes. (2 marks)
- (b) On **Figure 2**, on the opposite page, sketch the curve with equation $y = f(|x|)$, indicating the values where the curve cuts the coordinate axes. (3 marks)
- (c) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f\left(-\frac{1}{2}x\right)$. (4 marks)

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Answer space for question 7

(a)

$$y = -f(3x)$$

Figure 1


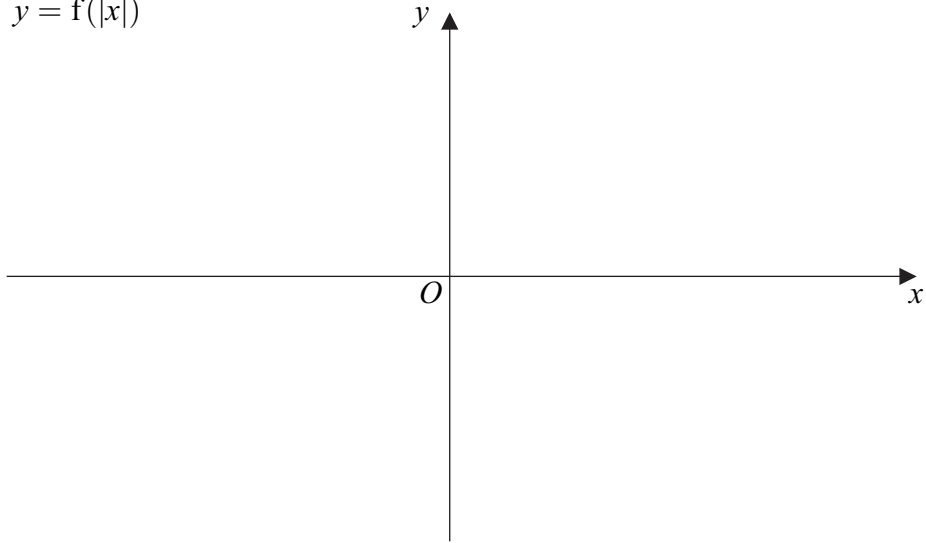
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(b)

$$y = f(|x|)$$

Figure 2

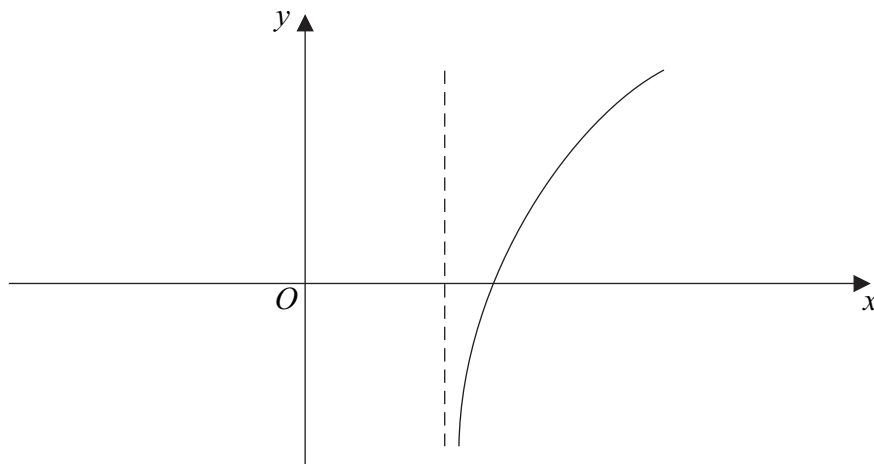


(c)

Area with horizontal dotted lines for writing answers.



- 8 The curve with equation $y = f(x)$, where $f(x) = \ln(2x - 3)$, $x > \frac{3}{2}$, is sketched below.



- (a) The inverse of f is f^{-1} .
- (i) Find $f^{-1}(x)$. (3 marks)
- (ii) State the range of f^{-1} . (1 mark)
- (iii) Sketch, on the axes given on the opposite page, the curve with equation $y = f^{-1}(x)$, indicating the value of the y -coordinate of the point where the curve intersects the y -axis. (2 marks)
- (b) The function g is defined by
- $$g(x) = e^{2x} - 4, \quad \text{for all real values of } x$$
- (i) Find $gf(x)$, giving your answer in the form $(ax - b)^2 - c$, where a , b and c are integers. (3 marks)
- (ii) Write down an expression for $fg(x)$, and hence find the exact solution of the equation $fg(x) = \ln 5$. (3 marks)

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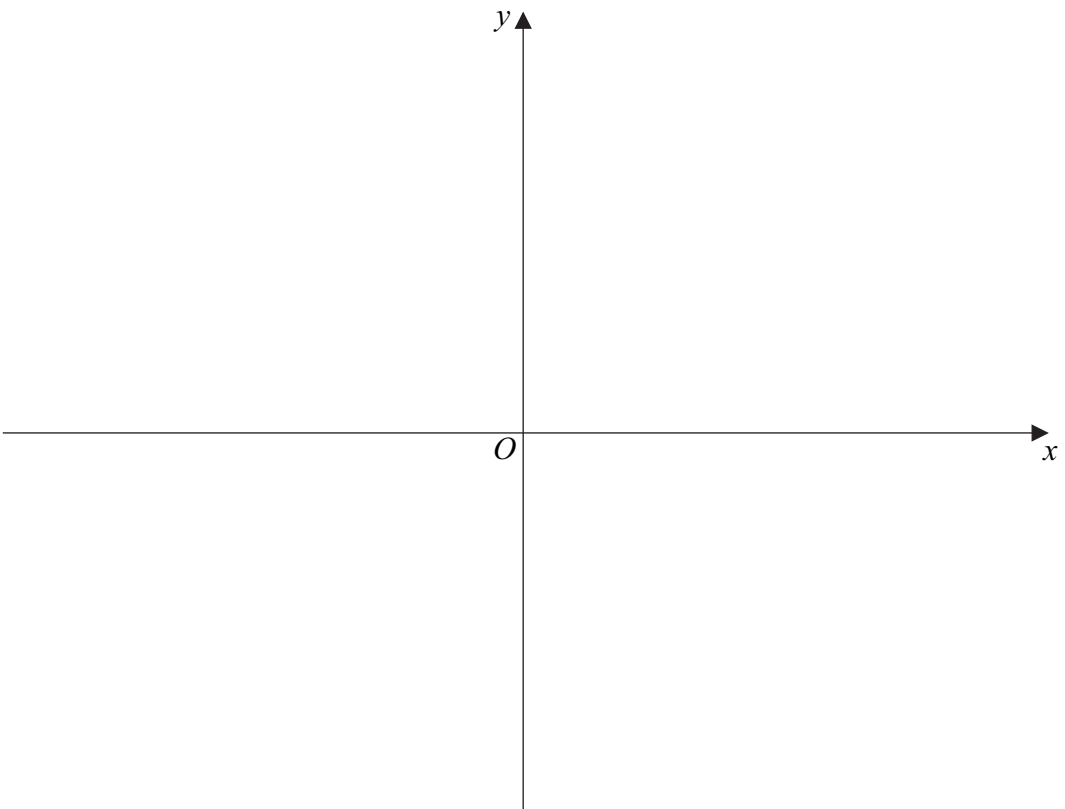
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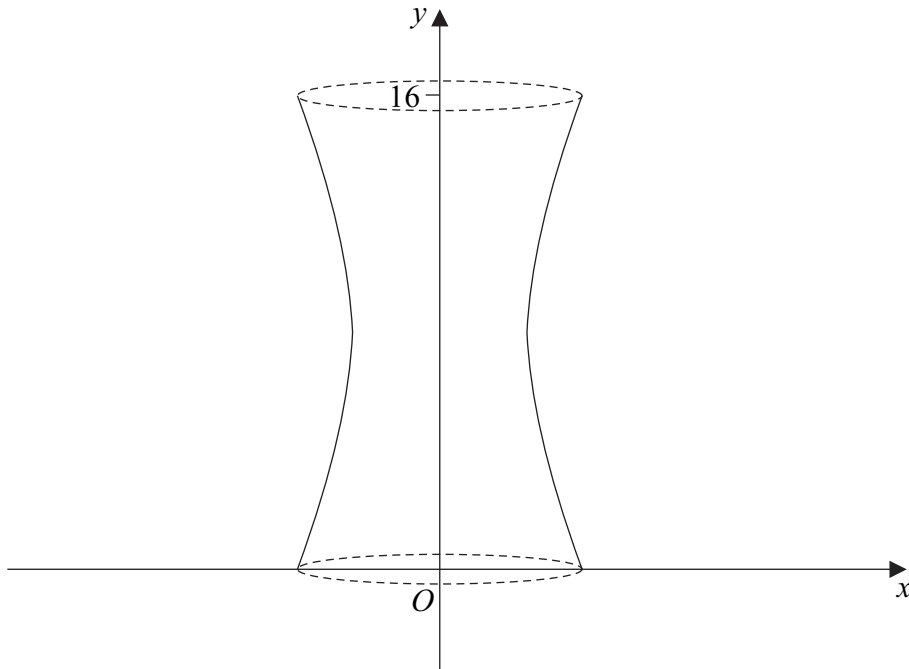
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9

The shape of a vase can be modelled by rotating the curve with equation $16x^2 - (y - 8)^2 = 32$ between $y = 0$ and $y = 16$ completely about the y -axis.



The vase has a base.

Find the volume of water needed to fill the vase, giving your answer as an exact value. (5 marks)

QUESTION PART REFERENCE

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10 (a) (i) By writing $\ln x$ as $(\ln x) \times 1$, use integration by parts to find $\int \ln x \, dx$. (4 marks)

(ii) Find $\int (\ln x)^2 \, dx$. (4 marks)

(b) Use the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{1}{x + \sqrt{x}} \, dx \quad (7 \text{ marks})$$

QUESTION
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Answer space for question 10



QUESTION
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Answer space for question 10

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QUESTION
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Answer space for question 10

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END OF QUESTIONS

