Full Solutions

Centre Number		Candidate Number	
Surname			
Other Names			
Candidate Signature			

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MPC3

Unit Pure Core 3

Monday 13 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

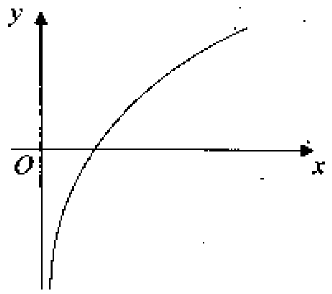
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The diagram shows the curve with equation $y = \ln(6x)$.



(a) State the x-coordinate of the point of intersection of the curve with the x-axis. *(1 mark)*

(b) Find $\dfrac{dy}{dx}$. *(2 marks)*

(c) Use Simpson's rule with 6 strips (7 ordinates) to find an estimate for $\displaystyle\int_1^7 \ln(6x)\,dx$, giving your answer to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

a) $6x = 1$ $\quad x = \dfrac{1}{6}$ B1

b) $y = \ln(6x)$

$\dfrac{dy}{dx} = \dfrac{6}{6x}$

$\dfrac{dy}{dx} = \dfrac{1}{x}$ M1 A1

QUESTION PART REFERENCE

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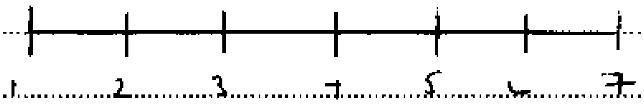
c)



$h = 1$ B1

x	$y = \ln(6x)$		
1	1.79176		
2		4(2.48491)	
3			2(2.89037)
4		4(3.17805)	
5			2(3.40120)
6		4(3.58352)	
7	3.73767		

M1

A1

$\frac{h}{3}(55.09848504)$

A1

$\underline{\underline{18.4}}$

7

Turn over ▶

Do not write outside the box

2 (a) (i) Find $\frac{dy}{dx}$ when $y = xe^{2x}$. *(3 marks)*

(ii) Find an equation of the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$. *(2 marks)*

(b) Given that $y = \frac{2\sin 3x}{1 + \cos 3x}$, use the quotient rule to show that

$$\frac{dy}{dx} = \frac{k}{1 + \cos 3x}$$

where k is an integer. *(4 marks)*

QUESTION PART REFERENCE

$y = [x][e^{2x}]$

a) (i)

$\frac{dy}{dx} = [1][e^{2x}] + [x][2e^{2x}]$ M1 A1

$= e^{2x} + 2xe^{2x}$ A1

(ii) $x_1 = 1 \quad y_1 = e^2$

$m = 3e^2$ B1

$y - e^2 = 3e^2(x - 1)$

$y - e^2 = 3e^2x - 3e^2$

$y = 3e^2x - 2e^2$ B1

o.e.

Do not write outside the box

QUESTION PART REFERENCE

$$\frac{dy}{dx} = \frac{[6\cos 3x][1+\cos 3x] - [2\sin 3x][-3\sin 3x]}{[1+\cos 3x]^2}$$ M1 A1

$$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{[1+\cos 3x]^2}$$

$$= \frac{6[\cos 3x + 1]}{[1+\cos 3x]^2}$$ M1

$$= \frac{6}{[1+\cos 3x]}$$ A1

$k = 6$

O.E.

(9)

Do not write outside the box

3 The curve $y = \cos^{-1}(2x - 1)$ intersects the curve $y = e^x$ at a single point where $x = \alpha$.

(a) Show that α lies between 0.4 and 0.5. *(2 marks)*

(b) Show that the equation $\cos^{-1}(2x - 1) = e^x$ can be written as $x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$. *(1 mark)*

(c) Use the iteration $x_{n+1} = \frac{1}{2} + \frac{1}{2}\cos(e^{x_n})$ with $x_1 = 0.4$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

QUESTION PART REFERENCE

$f(x) = \cos^{-1}(2x-1) - e^x$

a) $f(0.4) = +0.28032955$

$f(0.5) = -0.07792494$ M1

Change of sign implies

α lies between 0.4 and 0.5 A1

b) $\cos^{-1}(2x-1) = e^x$

$2x - 1 = \cos(e^x)$

$2x = 1 + \cos(e^x)$ B1

$x = \frac{1}{2} + \frac{1}{2}\cos(e^x)$

0 6

Do not write outside the box

QUESTION PART REFERENCE

c) $x_1 = 0.4$

$x_2 = 0.539$ B1

$x_3 = 0.428$ B1

5

Turn over ▶

Do not write outside the box

4 (a) (i) Solve the equation $\operatorname{cosec}\theta = -4$ for $0° < \theta < 360°$, giving your answers to the nearest 0.1°. *(2 marks)*

(ii) Solve the equation

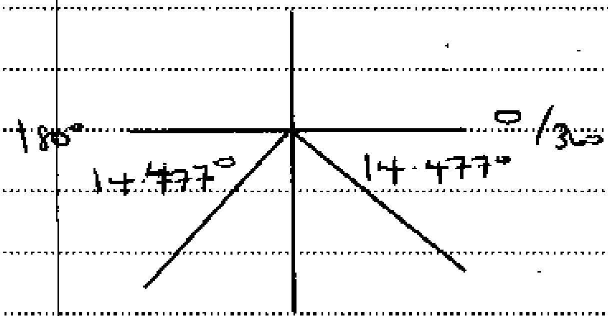
$$2\cot^2(2x + 30°) = 2 - 7\operatorname{cosec}(2x + 30°)$$

for $0° < x < 180°$, giving your answers to the nearest 0.1°. *(6 marks)*

(b) Describe a sequence of two geometrical transformations that maps the graph of $y = \operatorname{cosec} x$ onto the graph of $y = \operatorname{cosec}(2x + 30°)$. *(4 marks)*

QUESTION PART REFERENCE

a) (i) $\sin\theta = \frac{-1}{4}$



M1

$\theta = 194.5°\ ;\ 345.5°$ A1

(ii) $2\cot^2 Y = 2 - 7\operatorname{cosec} Y$ M1

$2 + 2\cot^2 Y = 4 - 7\operatorname{cosec} Y$ A1

$2\operatorname{cosec}^2 Y + 7\operatorname{cosec} Y - 4 = 0$

$(2\operatorname{cosec} Y - 1)(\operatorname{cosec} Y + 4) = 0$ A1

$(y = 2x + 30°)$

Do not write outside the box

QUESTION PART REFERENCE

$\operatorname{cosec} Y = \frac{1}{2}$ $\quad$ $\operatorname{cosec} Y = -4$

$Y = 194.5°, 345.5°$

$2x + 30° = 194.5 \,;\, 345.5°$ M1

$x = 82.2°, 157.8°$ A1

$\sin(2x + 30°) = 2$ HAS NO SOLUTIONS

$-1 \leq \sin \leq 1$ E1

b) HORIZONTAL TRANSLATION $\begin{bmatrix} -30° \\ 0 \end{bmatrix}$ B1

E1

STRETCH SCALE FACTOR $\times \frac{1}{2}$ E1

ALONG THE X- AXIS. E1

PARALLEL TO THE X-AXIS

(12)

Do not write outside the box

5 The functions f and g are defined with their respective domains by

$$f(x) = x^2 \qquad \text{for all real values of } x$$

$$g(x) = \frac{1}{2x+1} \qquad \text{for real values of } x, \quad x \neq -0.5$$

(a) Explain why f does not have an inverse. *(1 mark)*

(b) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(3 marks)*

(c) State the range of g^{-1}. *(1 mark)*

(d) Solve the equation $fg(x) = g(x)$. *(3 marks)*

QUESTION PART REFERENCE

a) IT IS NOT A ONE-TO-ONE FUNCTION E1

b) g(x) ×2
+1
Recip.

$g^{-1}(x) = \dfrac{\frac{1}{x} - 1}{2}$ M1 A1

$g^{-1}(x) = \dfrac{1}{2x} - \dfrac{1}{2}$ A1

c) $g^{-1}(x) \in \mathbb{R} \qquad g^{-1}(x) \neq -0.5$ E1

QUESTION PART REFERENCE

$f[g(x)] = g(x)$

d) $\left\{\frac{1}{2x+1}\right\}^2 = \frac{1}{2x+1}$ M1

A1

$2x+1 = 1$

$\underline{\underline{x = 0}}$ A1

(Given $x \neq -\frac{1}{2}$)

8

Turn over ▶

Do not write outside the box

6 (a) Given that $3\ln x = 4$, find the exact value of x. *(1 mark)*

(b) By forming a quadratic equation in $\ln x$, solve $3\ln x + \dfrac{20}{\ln x} = 19$, giving your answers for x in an exact form. *(5 marks)*

QUESTION PART REFERENCE

a) $3\ln x = 4$

$\ln x = \dfrac{4}{3}$

$x = e^{\frac{4}{3}}$ B1

b) $\times \ln x$ $\quad 3(\ln x)^2 + 20 = 19(\ln x)$

$3(\ln x)^2 - 19\ln x + 20 = 0$ M1

A1

$(3\ln x - 4)(\ln x - 5) = 0$

M1

$3\ln x = 4 \qquad \ln x = 5$

$x = e^{\frac{4}{3}} \qquad x = e^{5}$ A1

A1

6

Do not write outside the box

7 (a) On separate diagrams:

(i) sketch the curve with equation $y = |3x + 3|$; *(2 marks)*

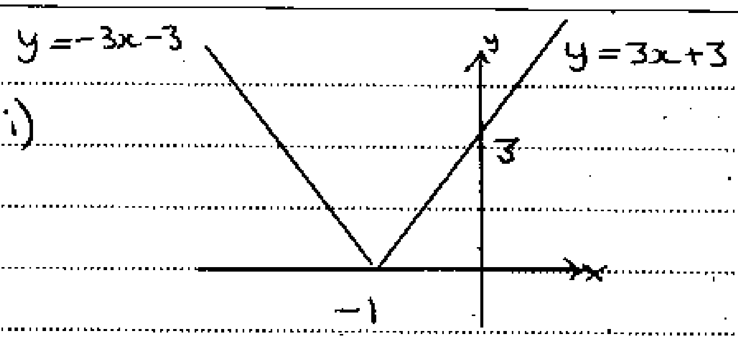
(ii) sketch the curve with equation $y = |x^2 - 1|$. *(3 marks)*

(b) (i) Solve the equation $|3x + 3| = |x^2 - 1|$. *(5 marks)*

(ii) Hence solve the inequality $|3x + 3| < |x^2 - 1|$. *(2 marks)*

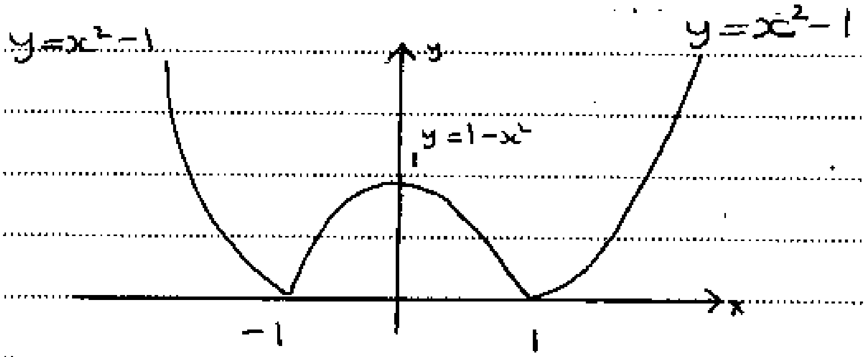
QUESTION PART REFERENCE

a) (i)



G2

(ii)



G3

b) (i) $3x + 3 = 1 - x^2$

$x^2 + 3x + 2 = 0$ M1

$(x + 2)(x + 1) = 0$

$\underline{\underline{x = -2 \quad x = -1}}$ A1

$3x + 3 = x^2 - 1$

$x^2 - 3x - 4 = 0$ M1

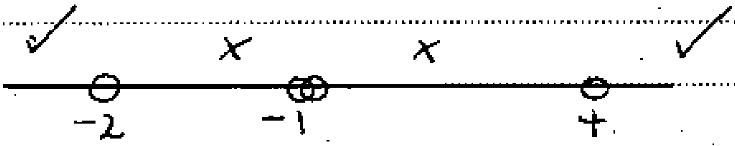
$(x - 4)(x + 1) = 0$ A1

$\underline{\underline{x = 4 \quad x \; -1}}$

$\underline{x = -2;\ x = -1 \quad x = 4}$ A1

QUESTION PART REFERENCE

(ii)



TEST $x = -3$ $6 < 8$ ✓

TEST $x = -1.5$ $3/2 < 5/4$ ✗

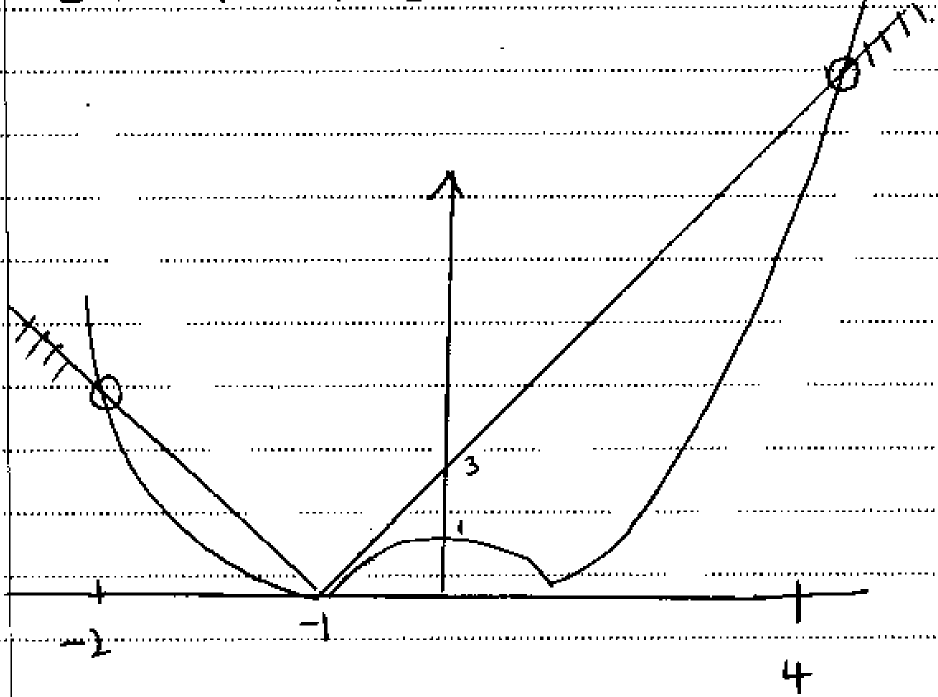
TEST $x = 0$ $3 < -1$ ✗

TEST $x = 5$ $18 < 24$ ✓

$x < -2$ $x > 4$

B2

OR VIA A SKETCH



(12)

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8 Use the substitution $u = 1 + 2\tan x$ to find

$$\int \frac{1}{(1+2\tan x)^2 \cos^2 x}\,dx$$

(5 marks)

QUESTION PART REFERENCE

$u = 1 + 2\tan x$ ①

$1\,du = 2\sec^2 x\,dx$ ②

$1\,du = \frac{2}{\cos^2 x}\,dx$

B1

$\frac{1}{2}\int \frac{1}{(1+2\tan x)^2}\,2\sec^2 x\,dx$

M1

A1

$\frac{1}{2}\int \frac{1}{u^2}\,du$

$-\frac{1}{2}u^{-1} + c$

M1

$-\frac{1}{2(1+2\tan x)} + C$

A1

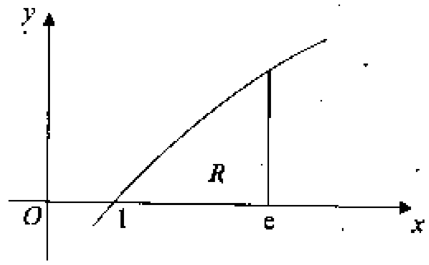
5

Do not write outside the box

9 (a) Use integration by parts to find $\int x \ln x \, dx$. *(3 marks)*

(b) Given that $y = (\ln x)^2$, find $\frac{dy}{dx}$. *(2 marks)*

(c) The diagram shows part of the curve with equation $y = \sqrt{x} \ln x$.



The shaded region R is bounded by the curve $y = \sqrt{x} \ln x$, the line $x = e$ and the x-axis from $x = 1$ to $x = e$.

Find the volume of the solid generated when the region R is rotated through 360° about the x-axis, giving your answer in an exact form. *(6 marks)*

QUESTION PART REFERENCE

a)

$\ln x$	$\frac{1}{2}x^2$
$\frac{1}{x}$	x

M1

$$\frac{1}{2}x^2 \ln x - \int \frac{1}{2}x \, dx$$ A1

$$\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$$ A1

QUESTION PART REFERENCE

Do not write outside the box

b) $\dfrac{dy}{dx} = \dfrac{2}{x}\ln x$ M1 A1

c) $\pi \displaystyle\int_1^e x(\ln x)^2\,dx$ B1

$(\ln x)^2$	$\frac{1}{2}x^2$
$\frac{2}{x}\ln x$	x

M1

$\frac{1}{2}x^2(\ln x)^2 - \int x\ln x\,dx \quad \times\pi$ A1

$\left[\frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x + \frac{1}{4}x^2\right]_1^e \quad \times\pi$ M1

$\left(\frac{1}{2}e^2 - \frac{1}{2}e^2 + \frac{1}{4}e^2\right) - \left(\frac{1}{4}\right) \quad \times\pi$ A1

$\left(\frac{1}{4}e^2 - \frac{1}{4}\right)\pi$ A1

(11)