MPC3 – June 2011

Q	Solution	Marks	Total	Comments
1 (a)	$\frac{1}{6}$ or $\left(\frac{1}{6}, 0\right)$	B1	1	condone 0.167 AWRT
(b)	$\left(\frac{dy}{dx}\right) = \frac{1}{x}$	M1		$\frac{k}{x}$ where $k = 1, 6$ or $\frac{1}{6}$
		A1	2	$k = 1$
(c)	x : y 1 : $\ln 6 = 1.7918$ 2 : $\ln 12 = 2.4849$ 3 : $\ln 18 = 2.8904$ 4 : $\ln 24 = 3.1781$ 5 : $\ln 30 = 3.4012$ 6 : $\ln 36 = 3.5835$ 7 : $\ln 42 = 3.7377$	M1		5+ y-values correct, either exact or correct to 3SF (rounded or truncated) or better
		A1		all 7 y-values correct (and only these 7 values), either exact or correct to 3SF (rounded or truncated) or better
	$A = \frac{1}{3} \times 1\left[(1.7918 + 3.7377) + 4(2.4849 + 3.1781 + 3.5835) + 2(2.8904 + 3.4012)\right]$	M1		correct use of Simpson's rule on their 7 y-values, condone missing square brackets
	$= 18.4$	A1	4	CAO this value only
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$y = xe^{2x}$			
	$\left(\frac{dy}{dx} =\right) 2xe^{2x} + e^{2x}$	M1 A1 A1 ISW	3	$kxe^{2x} + le^{2x}$ where k and l are 1s or 2s $k = 2$, $l = 1$ } Independent of each other $(= e^{2x}(2x+1))$
(ii)	$x = 1 \Rightarrow \frac{dy}{dx} = 3e^2$	M1		correct substitution of $x = 1$ into their $\frac{dy}{dx}$ but must have earned M1 in part (i)
	tangent: $y - e^2 = 3e^2(x-1)$ OE	A1	2	CSO (no ISW), must have scored first 4 marks common correct answer: $y = 3e^2x - 2e^2$
(b)	$y = \frac{2\sin 3x}{1+\cos 3x}$			
	$\left(\frac{dy}{dx} =\right) \frac{(1+\cos 3x)6\cos 3x - 2\sin 3x(-3\sin 3x)}{(1+\cos 3x)^2}$	M1		$\frac{\pm p(1+\cos 3x)\cos 3x \pm q\sin 3x(\sin 3x)}{(1+\cos 3x)^2}$ where p and q are rational numbers condone poor use/omission of brackets PI by further working
	$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$	A1		this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$), but allow $\sin^2 3x$ replaced by $1-\cos^2 3x$ condone denominator correctly expanded
		m1		correct use of $k\sin^2 3x + k\cos^2 3x = k$ or $k\sin^2 3x = k(1-\cos^2 3x)$
	$= \frac{6\cos 3x + 6}{(1+\cos 3x)^2}$			
	$= \frac{6}{1+\cos 3x}$	A1	4	CSO
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
	note: if degrees used then no marks in (a) and (c)			
3(a)	$f(x) = \cos^{-1}(2x-1) - e^x$			or reverse
	$f(0.4) = 0.3$ $f(0.5) = -0.1$	M1		sight of ± 0.3 (AWRT) **AND** ∓ 0.1 (AWRT)
	change of sign $\therefore 0.4 < \alpha < 0.5$	A1	2	CSO, note f (x) must be defined, condone $0.4 \le \alpha \le 0.5$
		(M1)		alternative method $e^{0.4} = 1.5$, $\cos^{-1}(2\times0.4 - 1) = 1.8$ $e^{0.5} = 1.65$, $\cos^{-1}(2\times0.5 - 1) = 1.57$
		(A1)		at 0.4 $e^x < \cos^{-1}(2x-1)$ at 0.5 $e^x > \cos^{-1}(2x-1)$ $\therefore 0.4 < \alpha < 0.5$
(b)	$\cos^{-1}(2x-1) = e^x$ $2x - 1 = \cos(e^x)$ $x = \frac{1}{2}\left(\cos(e^x) + 1\right) = \frac{1}{2} + \frac{1}{2}\cos(e^x)$	B1	1	AG must see middle line, and no errors seen, but condone cos e^x
(c)	$x_1 = 0.4$ $x_2 = 0.539$	B1		CAO
	$x_3 = 0.428$	B1	2	CAO
	Total		**5**	

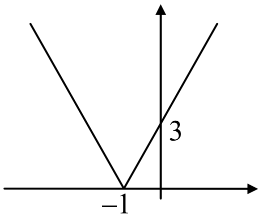
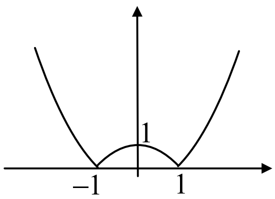
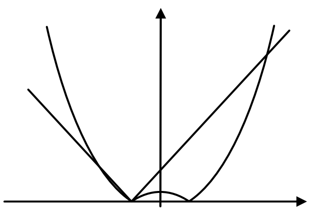
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\left(\sin^{-1} \pm 0.25 =\right) \pm 14.5$	M1		PI by sight of 194.5 etc condone ± 14.4
	$\theta = 194.5,\ 345.5$ (AWRT)	A1	2	no extras in interval, ignore answers outside interval
(ii)	$2\cot^2(2x+30) = 2 - 7\,\text{cosec}(2x+30)$			condone replacing $2x + 30$ by Y
	$2(\text{cosec}^2(2x+30) - 1) = 2 - 7\,\text{cosec}(2x+30)$	M1		correct use of $\text{cosec}^2 Y = 1 + \cot^2 Y$
	$2\,\text{cosec}^2(2x+30) + 7\,\text{cosec}(2x+30) - 4 (= 0)$	A1		must be in this form
	$(2\,\text{cosec}(2x+30) \pm 1)(\text{cosec}(2x+30) \pm 4)(= 0)$	m1		attempt at factorisation
	$\text{cosec}(2x+30) = \frac{1}{2}$ or -4	A1		must be this line using f $(2x + 30)$
	$2x + 30 = 194.5,\ 345.5$			
	$x = 82.2,\ 157.8$ (AWRT)	B1		one correct answer, allow 82.3, ignore extra solutions
		B1	6	CAO both answers correct and no extras in interval, ignore answers outside interval
(b)	stretch (I) scale factor $\frac{1}{2}$ (II) parallel to x-axis (III)	M1 A1		I and either II or III I + II + III
	translate	E1		
	$\begin{pmatrix} -15 \\ 0 \end{pmatrix}$	B1	4	condone '15 to left' or '–15 in x (direction)'
	alternative method translate	(E1)		
	$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$	(B1)		
	stretch	(M1)		as above
	scale factor $\frac{1}{2}$ parallel to x-axis	(A1)		as above
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\left[\mathrm{f}(x)\right]$ not 1 – 1	E1	1	OE
(b)	$y=\dfrac{1}{2x+1}$			
	$x=\dfrac{1}{2y+1}$	M1		swap x and y } either order
	$2y+1=\dfrac{1}{x}$	M1		a correct next line } either order
	$\left[g^{-1}(x)=\right]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$ OE	A1	3	$[y=]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$
(c)	$\left[\mathrm{g}^{-1}(x)\right]\neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left(\dfrac{1}{2x+1}\right)^2=\dfrac{1}{2x+1}$	B1		sight of $\left(\dfrac{1}{2x+1}\right)^2$ or $\dfrac{1}{(2x+1)^2}$
	$(2x+1)=(2x+1)^2$ or $2x+1=4x^2+4x+1$ or $\dfrac{1}{2x+1}=1$ or $2x+1=1$	M1		one correct step, must be one of these four lines
	$x=0$	A1	3	CSO
	Total		**8**	
6(a)	$3\ln x=4$ $\left(\ln x=\dfrac{4}{3}\right)$ $x=\mathrm{e}^{\frac{4}{3}}$	B1	1	ISW. Condone $\sqrt[3]{\mathrm{e}^4}$
(b)	$3\ln x+\dfrac{20}{\ln x}=19$			
	$3(\ln x)^2+20=19\ln x$	M1		correctly multiplying by $\ln x$.
	$3(\ln x)^2-19\ln x+20(=0)$	A1		
	$(3\ln x\pm 4)(\ln x\pm 5)(=0)$	m1		use of formula, or completing the square must be correct
	$\ln x=\dfrac{4}{3},\ 5$	A1		
	$x=\mathrm{e}^{\frac{4}{3}},\ \mathrm{e}^5$	A1	5	condone $\sqrt[3]{\mathrm{e}^4}$
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)		M1 A1	 2	modulus graph, approximate V shape, touching negative x-axis and crossing y-axis $-1, 3$ marked, graph symmetrical, straight lines
(ii)		M1 A1 A1	 3	modulus graph in 3 sections, touching x-axis and crossing positive y-axis correct curvature their $x > 1$, their $x < -1$ } independent correct curve $-1 \le x \le 1$ and $x = \pm 1$, $y = 1$ marked } independent
(b)(i)	$\|3x+3\| = \|x^2 - 1\|$ $(3x+3 = x^2 - 1)$ $(0 =)\ x^2 - 3x - 4$ —A $x = 4, -1$ $(3x + 3 = 1 - x^2)$ $x^2 + 3x + 2\ (= 0)$ —B $x = -1, -2$	 M1 A1,A1 A1,A1	 5	 either A or B seen, all terms on one side $\therefore\ x = -2, -1, 4$ SC NMS or partial method 1 correct value 1/5 2 correct values 2/5 3 correct values 5/5 } independent of method mark more than 3 distinct values max 2/5
(ii)	 $x > 4,\ \ x < -2$	M1,A1	2	$x >$ their largest, $x <$ their smallest; CAO
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8	$\int \frac{1}{\cos^2 x(1+2\tan x)^2}\,dx$			
	$u = 1 + 2\tan x$			
	$\left(\frac{du}{dx} =\right) 2\sec^2 x$ OE	M1		condone $\left(\frac{du}{dx} =\right) a\sec^2 x$ where a is a constant
	$\int = \int \frac{du}{2u^2}$	m1		$\int \frac{k}{u^2}\,(du)$, where k is a constant
		A1		correct, or $\frac{1}{2}\int u^{-2}\,(du)$
	$= \frac{1}{2}\frac{u^{-1}}{-1}$	A1F		correct integral of their expression but must have scored M1 m1
	$= -\frac{1}{2u}$			
	$= -\frac{1}{2(1+2\tan x)}(+c)$	A1	5	CSO, no ISW
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9 (a)	$\int x\ln x\,dx$			
	$u=\ln x \quad \frac{dv}{(dx)}=x$; $\frac{du}{(dx)}=\frac{1}{x} \quad v=\frac{x^2}{2}$	M1		correct direction and sight of $\frac{1}{x}$, $\frac{x^2}{2}$
	$\int=\frac{x^2}{2}\ln x-\int\frac{x^2}{2}\times\frac{1}{x}\,(dx)$	A1		
	$=\frac{x^2}{2}\ln x-\frac{x^2}{4}(+c)$	A1	3	
(b)	$y=(\ln x)^2$			
	$\left(\frac{dy}{dx}\right)=2\ln x\times\frac{1}{x}$	M1		$\frac{k}{x}\ln x$ where $k=\frac{1}{2}$, 1 or 2
		A1	2	$k=2$
(c)	$y=\sqrt{x}\,\ln x$			
	$(V=)\pi\int_1^e x(\ln x)^2\,dx$	B1		all correct, incl brackets, π, limits and dx (but dx may be seen BEFORE this line)
	$u=(\ln x)^2 \quad \frac{dv}{(dx)}=x$; $\frac{du}{(dx)}=2\ln x\frac{1}{x} \quad v=\frac{x^2}{2}$	M1		correct direction with $\frac{du}{(dx)}=\frac{k}{x}\ln x$ where $k=\frac{1}{2}$, 1 or 2 and sight of $\frac{x^2}{2}$
	$\int=\frac{x^2}{2}(\ln x)^2-\int\frac{x^2}{2}\times\frac{2}{x}\ln x\,(dx)$	m1		correct substitution of their terms into the parts formula
	$=\frac{x^2}{2}(\ln x)^2-\int x\ln x\,(dx)$	A1		integral needs to be simplified to $\int x\ln x$
	$=\frac{x^2}{2}(\ln x)^2-\frac{1}{4}x^2(2\ln x-1)$ OE			
	$V=(\pi)\left[\frac{x^2}{2}(\ln x)^2-\frac{1}{4}x^2(2\ln x-1)\right]_1^e$			
	$=(\pi)\left[\left(\frac{e^2}{2}-\frac{1}{4}e^2\right)-\left(0+\frac{1}{4}\right)\right]$	m1		correct substitution of 1 and e into their expressions of the form $px^2(\ln x)^2+qx^2\ln x+rx^2$ where p, q and r are non-zero rational numbers, and an intention to subtract. Do not condone F(1) – F(e)
	$=\frac{\pi}{4}\left[e^2-1\right]$ **OE**	A1	6	$\pi\left[\frac{e^2}{4}-\frac{1}{4}\right]$ etc
	Total		**11**	
	TOTAL		**75**	