Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's U
Examiner's Initials

Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MPC3

Unit Pure Core 3

Friday 11 June 2010 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The curve $y = 3^x$ intersects the curve $y = 10 - x^3$ at the point where $x = \alpha$.

(a) Show that α lies between 1 and 2. *(2 marks)*

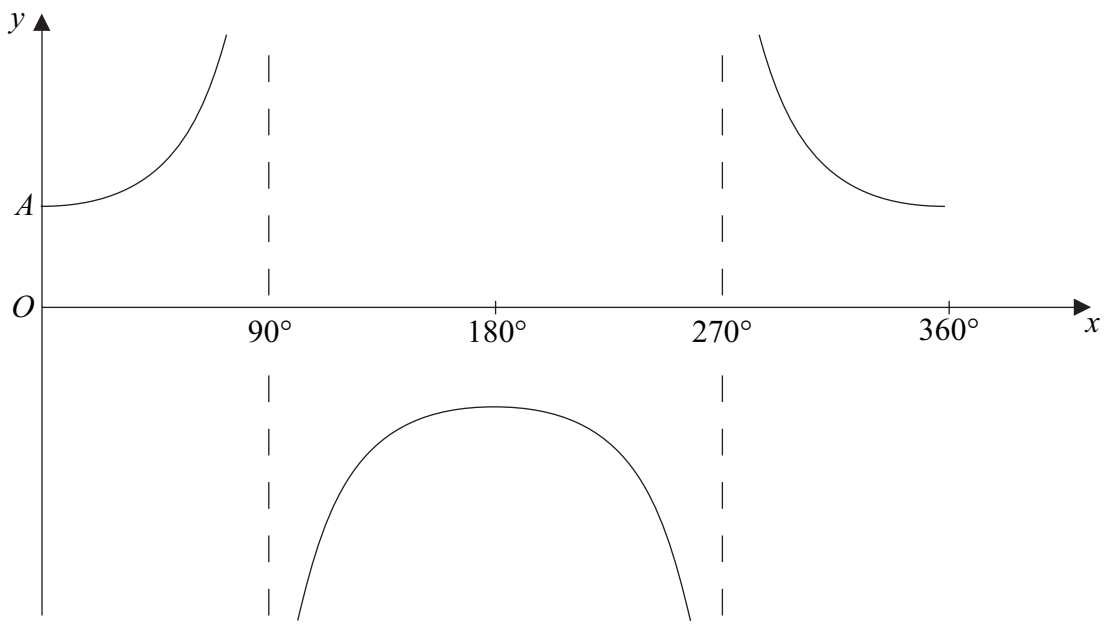
(b) (i) Show that the equation $3^x = 10 - x^3$ can be rearranged into the form $x = \sqrt[3]{10 - 3^x}$. *(1 mark)*

(ii) Use the iteration $x_{n+1} = \sqrt[3]{10 - 3^{x_n}}$ with $x_1 = 1$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

2 (a) The diagram shows the graph of $y = \sec x$ for $0° \leqslant x \leqslant 360°$.



(i) The point A on the curve is where $x = 0$. State the y-coordinate of A. *(1 mark)*

(ii) Sketch, on the axes given on page **5**, the graph of $y = |\sec 2x|$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

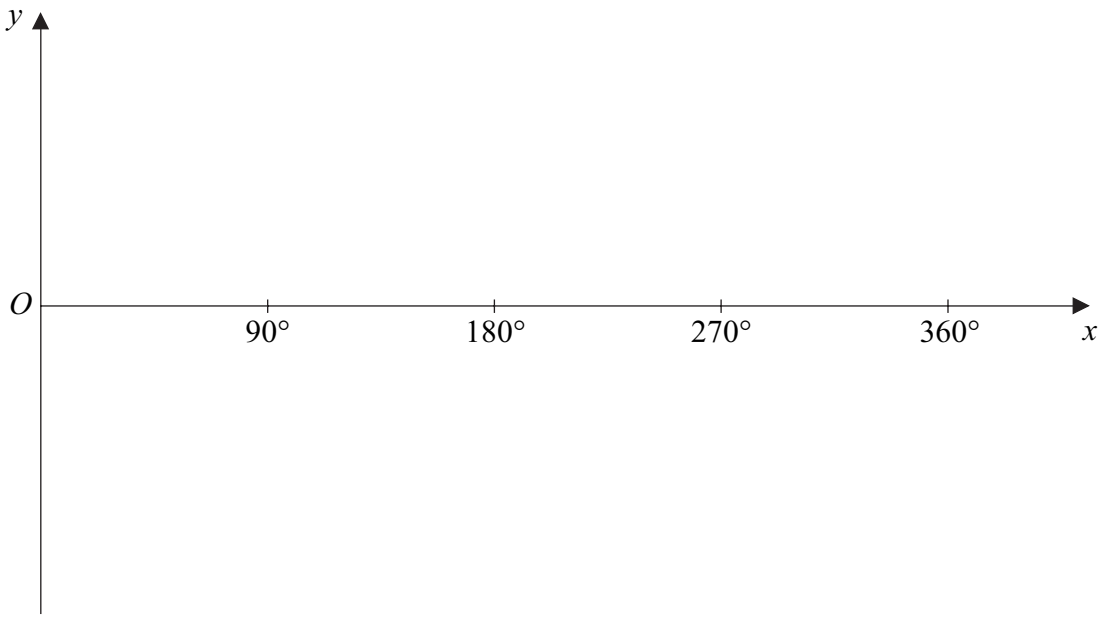
(b) Solve the equation $\sec x = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) Solve the equation $|\sec(2x - 10°)| = 2$, giving all values of x in degrees in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

(a)(ii)



3 (a) Find $\frac{dy}{dx}$ when:

(i) $y = \ln(5x - 2)$; *(2 marks)*

(ii) $y = \sin 2x$. *(2 marks)*

(b) The functions f and g are defined with their respective domains by

$f(x) = \ln(5x - 2)$, for real values of x such that $x \geqslant \frac{1}{2}$

$g(x) = \sin 2x$, for real values of x in the interval $-\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{4}$

(i) Find the range of f. *(2 marks)*

(ii) Find an expression for $gf(x)$. *(1 mark)*

(iii) Solve the equation $gf(x) = 0$. *(3 marks)*

(iv) The inverse of g is g^{-1}. Find $g^{-1}(x)$. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

4 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an approximation to $\int_{0.5}^{2} \frac{x}{1+x^3}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) Find the exact value of $\int_{0}^{1} \frac{x^2}{1+x^3}\,\mathrm{d}x$. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

5 (a) Show that the equation

$$10\,\text{cosec}^2 x = 16 - 11\cot x$$

can be written in the form

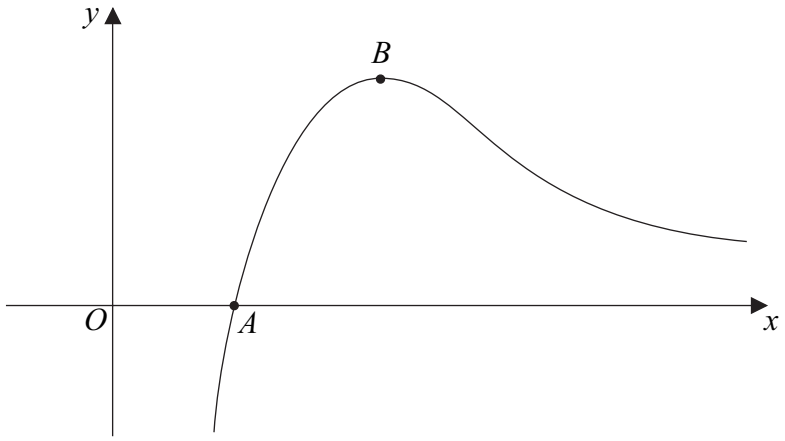
$$10\cot^2 x + 11\cot x - 6 = 0 \qquad \textit{(1 mark)}$$

(b) Hence, given that $10\,\text{cosec}^2 x = 16 - 11\cot x$, find the possible values of $\tan x$. *(4 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

6 The diagram shows the curve $y = \dfrac{\ln x}{x}$.



The curve crosses the x-axis at A and has a stationary point at B.

(a) State the coordinates of A. *(1 mark)*

(b) Find the coordinates of the stationary point, B, of the curve, giving your answer in an exact form. *(5 marks)*

(c) Find the exact value of the gradient of the normal to the curve at the point where $x = e^3$. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

7 (a) Use integration by parts to find:

(i) $\int x \cos 4x \, dx$; *(4 marks)*

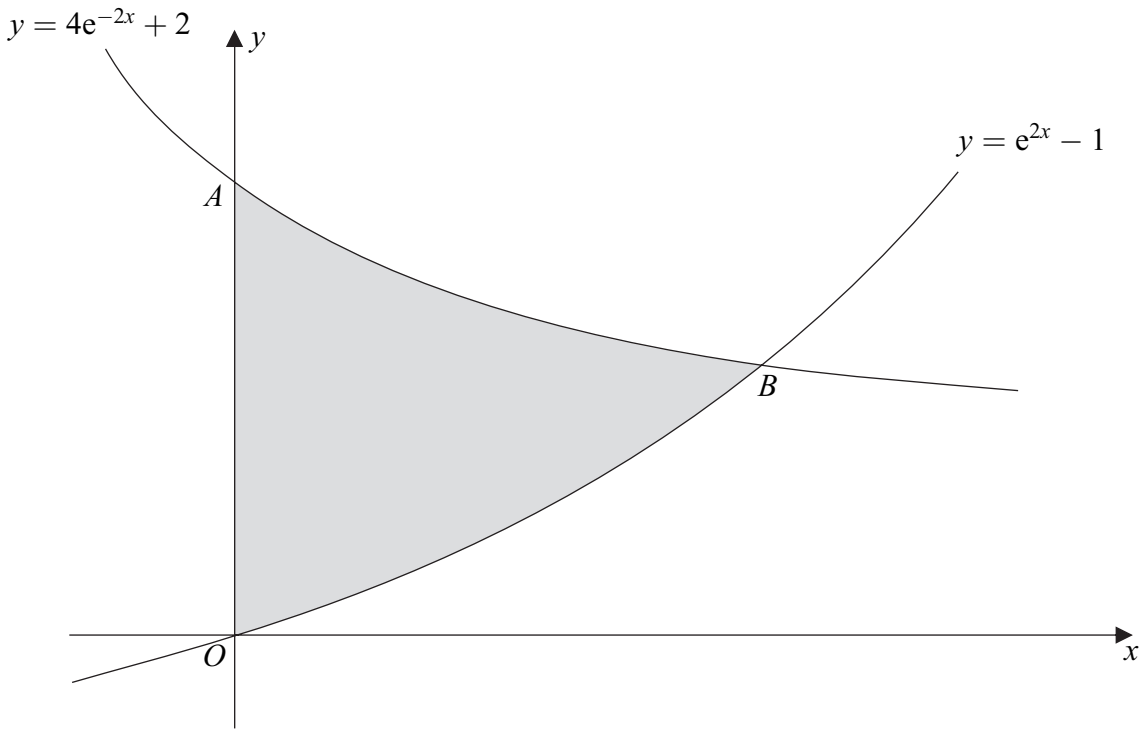
(ii) $\int x^2 \sin 4x \, dx$. *(4 marks)*

(b) The region bounded by the curve $y = 8x\sqrt{(\sin 4x)}$ and the lines $x = 0$ and $x = 0.2$ is rotated through 2π radians about the x-axis. Find the value of the volume of the solid generated, giving your answer to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

8 The diagram shows the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$.



The curve $y = 4e^{-2x} + 2$ crosses the y-axis at the point A and the curves intersect at the point B.

(a) Describe a sequence of two geometrical transformations that maps the graph of $y = e^{x}$ onto the graph of $y = e^{2x} - 1$. *(4 marks)*

(b) Write down the coordinates of the point A. *(1 mark)*

(c) (i) Show that the x-coordinate of the point B satisfies the equation

$$(e^{2x})^2 - 3e^{2x} - 4 = 0$$ *(2 marks)*

(ii) Hence find the exact value of the x-coordinate of the point B. *(3 marks)*

(d) Find the exact value of the area of the shaded region bounded by the curves $y = e^{2x} - 1$ and $y = 4e^{-2x} + 2$ and the y-axis. *(5 marks)*

QUESTION PART REFERENCE

.....

.....

.....

.....

.....

QUESTION PART REFERENCE

QUESTION PART REFERENCE	

QUESTION PART REFERENCE

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED