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General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

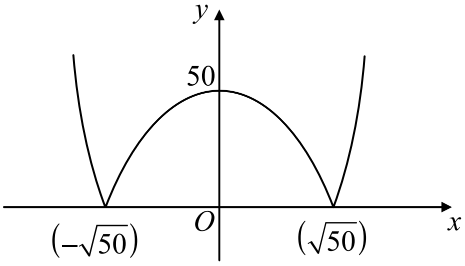
MPC3

Q	Solution	Marks	Total	Comments
1(a)(i)	$f(x)=\frac{\cos x}{2x+1}-\frac{1}{2}$			OE
	$f(0)=\frac{1}{2};\ f\left(\frac{\pi}{2}\right)=-\frac{1}{2}$	M1		$x=0$ LHS $=1$, $x=\frac{\pi}{2}$ LHS $=0$
	Change of sign $0<\alpha<\frac{\pi}{2}$	A1	2	Either side of $\frac{1}{2}$, $\therefore 0<\alpha<\frac{\pi}{2}$
(ii)	$\frac{\cos x}{2x+1}=\frac{1}{2}$			
	$\left.\begin{array}{l}2\cos x=2x+1\\2\cos x-1=2x\end{array}\right\}$ or, $\cos x=x+\frac{1}{2}$			Either line
	$x=\cos x-\frac{1}{2}$	B1	1	AG; or $\cos x-\frac{1}{2}=x$ All correct with no errors
(iii)	$x_1=0$			
	$x_2=0.5$	M1		Attempt at iteration $(\text{allow } x_2=-0.5, x_3=0.38, 0.4)$
	$x_3=0.378$	A1	2	CAO
(b)(i)	$\frac{dy}{dx}=\frac{(2x+1)(-\sin x)-\cos x\times 2}{(2x+1)^2}$	M1		Attempt at quotient rule: $\frac{\pm(2x+1)\sin x\pm 2\cos x}{(2x+1)^2}$
		A1		Either term correct
		A1	3	All correct ISW
(ii)	$x=0$			
	$\frac{dy}{dx}=-2$	m1		Correctly subst. $x=0$ into their $\frac{dy}{dx}$
	$\therefore$ Gradient of normal $=\frac{1}{2}$	A1	2	CSO
	Total		**10**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\mathrm{f}(x) \geqslant 0$	M1		For $\geqslant 0,\ \mathrm{f}(x) > 0$
		A1	2	Correct; allow $y \geqslant 0,\ \mathrm{f} \geqslant 0$
(b)(i)	$y = \sqrt{2x+5}$			
	$x = \sqrt{2y+5}$	M1		$x \Leftrightarrow y$
	$x^2 = 2y+5$	M1		Attempt to isolate, squaring first
	$\mathrm{f}^{-1}(x) = \dfrac{x^2-5}{2}$	A1	3	condone $(y = \ )$
(ii)	$x \geqslant 0$	B1F	1	ft their (a), but **must** be x
2(c)(i)	$\mathrm{h}(x) = \mathrm{fg}(x)$			
	$= \sqrt{2\left(\dfrac{1}{4x+1}\right)+5}$	B1	1	
(ii)	$\sqrt{2\left(\dfrac{1}{4x+1}\right)+5} = 3$			
	$2\left(\dfrac{1}{4x+1}\right)+5 = 9$	M1		one correct step from (c)(i), squaring
	$\left.\begin{array}{l}\dfrac{1}{4x+1} = 2 \\ 4x+1 = \dfrac{1}{2}\end{array}\right\}$ either or $16x+4=2$	A1		
	$x = -\dfrac{1}{8}$ or equiv	A1	3	CSO
	Total		**10**	
3(a)	$\tan^{-1}\left(-\dfrac{1}{3}\right) = -0.32$	M1		Sight of $\pm$ 0.32 or 18.43
		A1		a correct answer AWRT
	$x = 2.82, 5.96$	A1	3	-1 for any extra in range, ignore extra answers not in range. [SC 161.57, 341.57 AWRT M1A1 (max 2/3)]
(b)	$3\left(\tan^2 x + 1\right) = 5\tan x + 5$			
	$3\tan^2 x - 5\tan x - 2 = 0$	B1	1	AG
3(c)	$(3\tan x + 1)(\tan x - 2) = 0$	M1		Attempt at factorisation/formula
	$\tan x = 2, -\dfrac{1}{3}$	A1		
	$x = 1.11, 4.25, 2.82, 5.96$ AWRT	B1		3 correct [SC $x = 1.11, 4.25$ + their two answers from (a)]
		B1	4	4 correct, no extras in range [SC 161.57, 341.57, 63.43, 243.43 AWRT B1 (max 3/4)]
	Total		**8**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)		M1		Modulus graph, 3 section, condone shape inside + outside $\pm\sqrt{50}$
		A1		Cusps + curvature outside $\pm\sqrt{50}$
		A1	3	Value of y and shape inside $\left(\pm\sqrt{50}\right)$
(b)	$\left\lvert 50-x^2 \right\rvert = 14$			
	$50-x^2=14 \quad x^2=36$ $50-x^2=-14 \quad x^2=64$	M1		Either
	$x=\pm 6, \pm 8$	A1		2 correct, from correct working
		A1	3	All 4 correct, from correct working
(c)	$-6 < x < 6$	B1		
	$x > 8, \ x < -8$	B1	2	
(d)	Reflect in x-axis	M1,A1		or $\left\{\begin{matrix}\text{Reflect in } y=a \\ \text{Translate } \begin{bmatrix}0\\50-2a\end{bmatrix}\end{matrix}\right\}$
	Translate $\begin{bmatrix}0\\50\end{bmatrix}$	E1, B1	4	or $\left\{\begin{matrix}\text{Translate } \begin{bmatrix}0\\-50\end{bmatrix} \\ \text{Reflect in } x-\text{axis}\end{matrix}\right\}$ or $\left\{\begin{matrix}\text{Translate } \begin{bmatrix}0\\2a-50\end{bmatrix} \\ \text{Reflect in } y=a\end{matrix}\right\}$
	Reflect in $y = 25$ scores 4/4			
	Total		**12**	
5(a)	$2\ln x = 5$			
	$\ln x = \dfrac{5}{2} \quad x = \mathrm{e}^{\frac{5}{2}}$	B1	1	
(b)	$2\ln x + \dfrac{15}{\ln x} = 11$			
	$2(\ln x)^2 - 11\ln x + 15 = 0$	M1		Forming quadratic equation in $\ln x$, condone poor notation
	$(2\ln x - 5)(\ln x - 3) = 0$	m1		Attempt at factorisation/formula
	$\ln x = \dfrac{5}{2}, 3 \qquad$ condone $2\ln x = 5$	A1		
	$x = \mathrm{e}^{\frac{5}{2}}, \mathrm{e}^3$	A1,A1	5	[SC for substituting $x = \mathrm{e}^{\frac{5}{2}}$ or equivalent into equation and verifying B1 $\left(\frac{1}{5}\right)$]
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$V=\pi\int x^2\,\mathrm{d}y$	B1		PI
	$V=\frac{(\pi)}{4}\int\left(100-y^2\right)\mathrm{d}y$	M1		$k\int\left(100-y^2\right)\mathrm{d}y$ may be recovered Allow $\int\left(\text{their }x\right)^2\mathrm{d}y$, expanded
	$=\frac{(\pi)}{4}\left[100y-\frac{y^3}{3}\right]_{(0)}^{(10)}$	A1		
	$=\frac{(\pi)}{4}\left[\frac{2000}{3}\right]$	m1		For F(10) − F(0)
	$=\frac{500\pi}{3}$	A1	5	OE CSO
				SC: if rotated about x-axis $V=\pi\left[100x-\frac{4x^3}{3}\right]_0^5$ M1 $=\frac{1000}{3}\pi$ A1 max 2/5
(b)	x y 0.5 9.95(0) 1.5 9.539 2.5 8.66(0) 3.5 7.141 4.5 4.359 or better	B1 M1 A1		Correct x 4 + correct y to 2sf All y correct
	$A=1\times\sum y=39.6$	A1	4	(39.6 scores $4/4$)
6(c)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(100-4x^2\right)^{-\frac{1}{2}}(-8x)$	M1		Chain rule $(\ )^{-\frac{1}{2}}\times\mathrm{f}(x)$; allow f($x$) = k $\mathrm{f}(x)=\frac{1}{2}(-8x)=-4x$
	$x=3\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=-12\left(100-36\right)^{-\frac{1}{2}}$	A1		
	$=-\frac{3}{2}$ or equivalent	A1	3	CSO
(ii)	$y-8=-\frac{3}{2}(x-3)$	M1		$y-8=\left(\text{their }\frac{\mathrm{d}y}{\mathrm{d}x}\right)(x-3)$ or $y=\left(\text{their }\frac{\mathrm{d}y}{\mathrm{d}x}\right)x+c$ **and** subst. (3,8) to find c
	$\left(2y-16=-3x+9\right)$ $2y+3x=25$	A1	2	AG; all correct with no slips, full marks in part (i)

MPC3

Q	Solution	Marks	Total	Comments
6(d)	$x = 0 \quad y = \frac{25}{2}$ or equivalent	B1		
	$y = 0 \quad x = \frac{25}{3}$	B1		OE
	Area of $\Delta = \frac{1}{2}\times\frac{25}{2}\times\frac{25}{3}$	M1		for $\frac{1}{2}(\text{their } y)\times(\text{their } x)$ or $\frac{1}{2}ab\sin C$
	Area = Area Δ – (b)	m1		PI Δ > (b)
	Required area = 12.5 AWRT	A1	5	Condone 12.4 AWRT
(d)	**Alternative**			
	Area $\Delta = \int_{0}^{\frac{25}{3}} \frac{1}{2}(25-3x)(\mathrm{d}x)$	(B1) (B1)		
	$= \frac{1}{2}\left[25x - \frac{3x^2}{2}\right]_0^{\frac{25}{3}}$ $\frac{1}{2}\left[\frac{625}{3} - \frac{625}{6}\right]$	(M1)		For integration **and** $\mathrm{f}(\frac{25}{3}) - \mathrm{f}(0)$
	$= \frac{625}{12}$			
	Total		**19**	
7(a)	$\int (t-1)\ \ln t\ \mathrm{d}t$			
	$u = \ln t \quad \frac{\mathrm{d}v}{\mathrm{d}t} = t-1$	M1		Differentiate + integrate, correct direction
	$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{t} \quad v = \frac{t^2}{2} - t$	A1		All correct
	$\int = \left(\frac{t^2}{2} - t\right)\ln t - \int\left(\frac{t^2}{2} - t\right)\times\frac{1}{t}(\ \mathrm{d}t)$			
	$= \left(\frac{t^2}{2} - t\right)\ln t - \int\left(\frac{t}{2} - 1\right)(\ \mathrm{d}t)$	A1		Condone missing brackets
	$= \left(\frac{t^2}{2} - t\right)\ln t - \frac{t^2}{4} + t(+c)$	A1	4	CAO

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	**Alternative**			
	$\int(t-1)\ln t$	(M1)		$u=\ln t \quad v'=(t-1)$
		(A1)		$u'=\frac{1}{t} \quad v=\frac{(t-1)^2}{2}$
	$\int = \frac{(t-1)^2}{2}\ln t - \int\frac{(t-1)^2}{t}\,\frac{1}{t}\,\mathrm{d}t$			
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\int\frac{t^2-2t+1}{t}\,\mathrm{d}t$			
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\int t-2+\frac{1}{t}\,\mathrm{d}t$	(A1)		
	$\frac{(t-1)^2}{2}\ln t - \frac{1}{2}\left[\frac{t^2}{2}-2t+\ln t\right]$	(A1)		
	$=\frac{t^2}{2}\ln t - t\ln t + \cancel{\frac{1}{2}\ln t} - \frac{t^2}{4} + t - \cancel{\frac{1}{2}\ln t}$			
	$=\left(\frac{t^2}{2}-t\right)\ln t - \frac{1}{4}t^2 + t + c$		(4)	
(b)	$t=2x+1$			
	$\mathrm{d}t = 2\,\mathrm{d}x$ (RHS)	M1		$\frac{\mathrm{d}t}{\mathrm{d}x}=2$ (LHS)
	$2x=t-1,$	m1		OE
	$\int=\int \cancel{2}(t-1)\ln t\frac{\mathrm{d}t}{\cancel{2}}$	A1	3	AG
(c)	$[x]_0^1=[t]_1^3$	M1		Limit becoming 3
	$\int=\left[\left(\frac{t^2}{2}-t\right)\ln t-\frac{t^2}{4}+t\right]_1^3$			
	$=\left[\left(\frac{9}{2}-3\right)\ln 3-\frac{9}{4}+3\right]-\left[0-\frac{1}{4}+1\right]$	m1		Correctly sub. 1,3 into their (a)
	$=\frac{3}{2}\ln 3$	A1	3	CSO
	or			
	$\int=\left[\left(\frac{(2x+1)^2}{2}-(2x+1)\right)\ln(2x+1)-\frac{(2x+1)^2}{4}+(2x+1)\right]_0^1$	(M1)		Condone 1 slip
	$=\left(\left(\frac{9}{2}-3\right)\ln 3-\frac{9}{4}+3\right)-\left(0-\frac{1}{4}+1\right)$	(m1)		Correctly sub. 0,1
	$=\frac{3}{2}\ln 3$	(A1)	(3)	CSO
	Total		**10**	
	TOTAL		**75**	