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General Certificate of Education

Mathematics 6360

MPC3 **Pure Core 3**

Mark Scheme

2007 examination - June series

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Key to mark scheme and abbreviations used in marking

M	mark is for method		
m or dM	mark is dependent on one or more M marks and is for method		
A	mark is dependent on M or m marks and is for accuracy		
B	mark is independent of M or m marks and is for method and accuracy		
E	mark is for explanation		
√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
-x EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

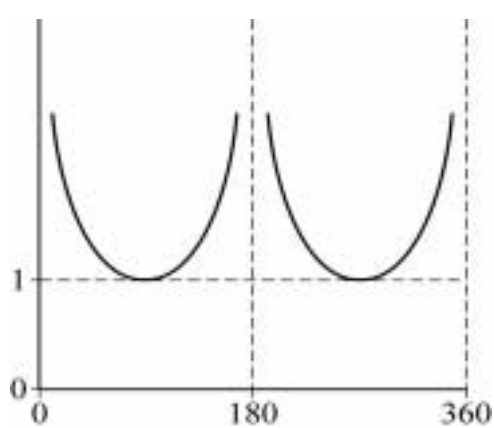
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

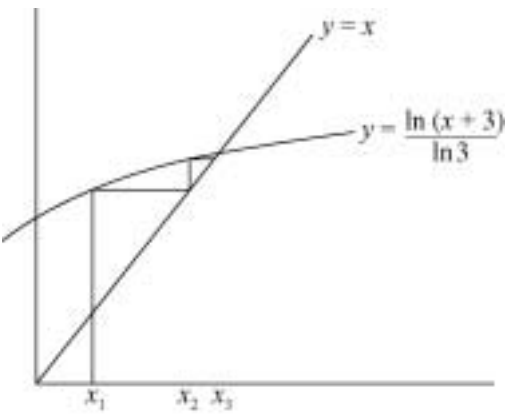
MPC3

Q	Solution	Marks	Total	Comments
1(a)	$y = \ln x$ $\frac{dy}{dx} = \frac{1}{x}$	B1	1	penalise + c once on 1(a) or 2(a)
(b)	$y = (x+1)\ln x$ $\frac{dy}{dx} = (x+1) \times \frac{1}{x} + \ln x$	M1 A1	2	product rule
(c)	$y = (x+1)\ln x$ $\frac{dy}{dx} = \frac{1}{x} + 1 + \ln x$ $x=1: \frac{dy}{dx} = 1+1=2$ Grad normal = $-\frac{1}{2}$ $y = -\frac{1}{2}(x-1)$	M1 M1 A1 A1	 4	substitute $x = 1$ into their $\frac{dy}{dx}$ use of $m_1 m_2 = -1$ CSO OE
Total			7	
2(a)	$4(x-1)^3$ or in expanded form	B1	1	allow $-4(1-x)^3$
(b)	$V = 4(\pi) \int_2^4 (x-1)^3 dx$ $= 4\pi \left[\frac{(x-1)^4}{4} \right]_2^4$ $= \pi(81-1) = 80\pi$	M1 M1 m1 A1	 4	$(\pi) \int y^2 dx$ $k(x-1)^4 (\pi)$ or in expanded form correct substitution of limits into $k(x-1)^4$ CAO
(c)	Translate $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ Stretch (I) SF 2 (II) // y axis (III)	E1 B1 M1 A1	 4	OE for I and (II or III) for I and II and III
Total			9	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\operatorname{cosec} x = 2$ $\Rightarrow \sin x = \frac{1}{2}$ $x = 30, 150$	M1 A1	2	30° scores M1 implied and no extras in range
(b)(i)	1	B1	1	
(ii)		M1 A1	2	all positive, 2 U shapes minima consistent > 0 , not intersecting with each other or y -axis
(c)	$x = 30, 150, 210, 330$	B1F B1	2	3 correct values from their (a), which must be $\theta, 180 - \theta$ all correct and no extras in range
	Total		7	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	y			
	x_0 1 3	B1		x values PI
	x_1 1.25 3.948(2)			
	x_2 1.5 5.196(2)	B1		(4+) y values correct
	x_3 1.75 6.838(5)			
	x_4 2 9			
	$A = \frac{1}{3} \times \frac{1}{4} (3 + 4 \times 3.9482 + 2 \times 5.1962$	M1		Simpson's rule
	$+ 4 \times 6.8385 + 9)$	A1	4	CAO
	$= 5.46$			
(b)(i)	$f(x) = 3^x - x - 3$ $f(0.5) = -1.77$ $f(1.5) = 0.696$ } change of sign \therefore root	M1A1	2	
(ii)	$3^x = x + 3$ $\ln 3^x = \ln(x + 3)$ $x \ln 3 = \ln(x + 3)$ $x = \frac{\ln(x + 3)}{\ln 3}$	M1 A1	 2	correct use of logs correct with no mistakes; AG
(iii)	$x_1 = 0.5$ $(x_2 = 1.14)$ $x_3 = 1.29 = 1.3$	M1 A1	 2	CAO
(iv)		M1 A1	 2	staircase x_2, x_3 correct and labelled on x -axis
Total			12	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$f(x) \geq 0$ allow $y \geq 0$	M1 A1	2	> 0 or $f \geq 0$ or ≥ 0
(b)(i)	$\sqrt{\frac{1}{x} - 2}$	B1	1	
(ii)	$\frac{1}{x} - 2 = 1$ $\frac{1}{x} = 3$ $x = \frac{1}{3}$	M1 A1 A1	3	squaring their (b)(i) in an equation CSO
(c)	$y = \sqrt{x-2}$ $y^2 = x-2$ $x^2 = y-2$ $y = x^2 + 2$	M1 M1 A1	3	attempt to isolate; condone 1 slip reverse $x \Leftrightarrow y$
Total			9	
6(a)	$\int xe^{5x} dx$ $u = x \quad dv = e^{5x}$ $du = 1 \quad v = \frac{1}{5}e^{5x}$ $\int = \frac{1}{5}xe^{5x} - \int \frac{1}{5}e^{5x} dx$ $= \frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} (+c)$	M1 A1 A1 A1	4	integrate one term, differentiate one term
(b)(i)	$u = x^{\frac{1}{2}}$ $du = \frac{1}{2}x^{-\frac{1}{2}} dx$ $\int = \int \frac{1}{1+u} \times 2 du$	M1 A1	2	correct with no errors; AG
(ii)	$\int_1^9 dx = \int_1^3 \frac{2}{1+u} du$ $= [2 \ln(1+u)]_1^3$ $= 2 \ln 4 - 2 \ln 2$ $(= \ln 4)$	m1 M1 A1	3	correct limits used in correct expression, ignoring k for $k \ln(1+u)$ ISW OE
Total			9	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$y = (x^2 - 3)e^x$ $\frac{dy}{dx} = (x^2 - 3)e^x + 2xe^x$	M1 A1	2	product rule
(ii)	$\frac{d^2y}{dx^2} = (x^2 - 3)e^x + 2xe^x + 2xe^x + 2e^x$	M1 A1	2	product rule from their $\frac{dy}{dx}$
(b)(i)	$\frac{dy}{dx} = 0$ $\Rightarrow e^x(x^2 + 2x - 3) = 0$ $e^x(x+3)(x-1) = 0$ $\therefore x = -3, 1$	M1 m1 A1 A1	4	$e^x f(x) = 0$ from $\frac{dy}{dx} = 0$ attempt at factorising or use of formula first correct solution second correct solution, and no others SC No working shown: $x = -3$ B2, $x = 1$ B2
(ii)	$x = -3$ $y'' = -4e^x$ max (-0.2) $x = 1$ $y'' = 4e^x$ min (10.9)	M1 A1	2	Condone slip
Total			10	
8(a)	$\tan x$ (+ c)	B1	1	
(b)	$f(x) = \frac{\cos x}{\sin x}$ $f'(x) = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$ $= \frac{-1}{\sin^2 x}$ $= -\operatorname{cosec}^2 x$	M1 A1 A1 A1	4	quotient rule $\frac{\pm \sin^2 x \pm \cos^2 x}{\sin^2 x}$ use of $\sin^2 x + \cos^2 x = 1$ AG CSO Special cases $f(x) = \frac{\cot x}{1}$ $f'(x) = \frac{1 \times -\operatorname{cosec}^2 x - \cot x \times 0}{1^2}$ M1 $= -\operatorname{cosec}^2 x$ A1 (max 2/4) Or $f(x) = \frac{1}{\tan x}$ $f'(x) = \frac{\tan x \times 0 - 1 \times \sec^2 x}{\tan^2 x}$ M1 A1 $= \frac{-\sec^2 x}{\tan^2 x}$ $= \frac{-1}{\sin^2 x} = -\operatorname{cosec}^2$ A1 (max 3/4)

MPC3 (cont)

Q	Solution	Marks	Total	Comments
(c)	$\begin{aligned} \text{LHS} &= \tan^2 x + \cot^2 x + 2 \tan x \cot x \\ &= \tan^2 x + 1 + \cot^2 x + 1 \\ &= \sec^2 x + \operatorname{cosec}^2 x \\ &= \text{RHS} \end{aligned}$	M1 M1 A1	3	expanding correct use of trig identities CSO
(d)	$\begin{aligned} \int (\tan x + \cot x)^2 dx &= \int \sec^2 x + \operatorname{cosec}^2 x dx \\ &= [\tan x - \cot x]_{0.5}^1 \\ &= 0.9153 - -1.2842 \\ &= 2.2 \end{aligned}$	M1 M1 A1 A1	4	use of identity $\pm \tan x \pm \cot x$ OE AWRT
	Total		12	
	TOTAL		75	