

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	



General Certificate of Education
Advanced Level Examination
January 2013

Mathematics

MPC3

Unit Pure Core 3

Wednesday 23 January 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

- Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
 - Fill in the boxes at the top of this page.
 - Answer **all** questions.
 - Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
 - You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
 - Do not write outside the box around each page.
 - Show all necessary working; otherwise marks for method may be lost.
 - Do all rough work in this book. Cross through any work that you do not want to be marked.

- Information**
- The marks for questions are shown in brackets.
 - The maximum mark for this paper is 75.

- Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
 - You do not necessarily need to use all the space provided.



Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Show that the equation $x^3 - 6x + 1 = 0$ has a root α , where $2 < \alpha < 3$. (2 marks)

(b) Show that the equation $x^3 - 6x + 1 = 0$ can be rearranged into the form

$$x^2 = 6 - \frac{1}{x} \quad (1 \text{ mark})$$

(c) Use the recurrence relation $x_{n+1} = \sqrt{6 - \frac{1}{x_n}}$, with $x_1 = 2.5$, to find the value of x_3 , giving your answer to four significant figures. (2 marks)

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2 (a) Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^4 \frac{x}{x^2 + 2} dx$$

Give your answer to four significant figures. (4 marks)

(b) Show that the exact value of $\int_0^4 \frac{x}{x^2 + 2} dx$ is $\ln k$, where k is an integer. (5 marks)

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3 (a) Find $\frac{dy}{dx}$ when

$$y = e^{3x} + \ln x \quad (2 \text{ marks})$$

(b) (i) Given that $u = \frac{\sin x}{1 + \cos x}$, show that $\frac{du}{dx} = \frac{1}{1 + \cos x}$. (3 marks)

(ii) Hence show that if $y = \ln\left(\frac{\sin x}{1 + \cos x}\right)$, then $\frac{dy}{dx} = \operatorname{cosec} x$. (2 marks)

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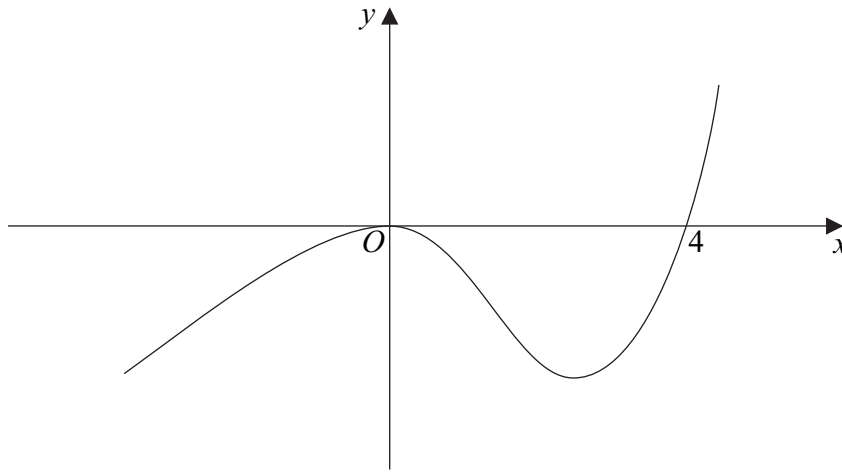
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4

The diagram shows a sketch of the curve with equation $y = f(x)$.

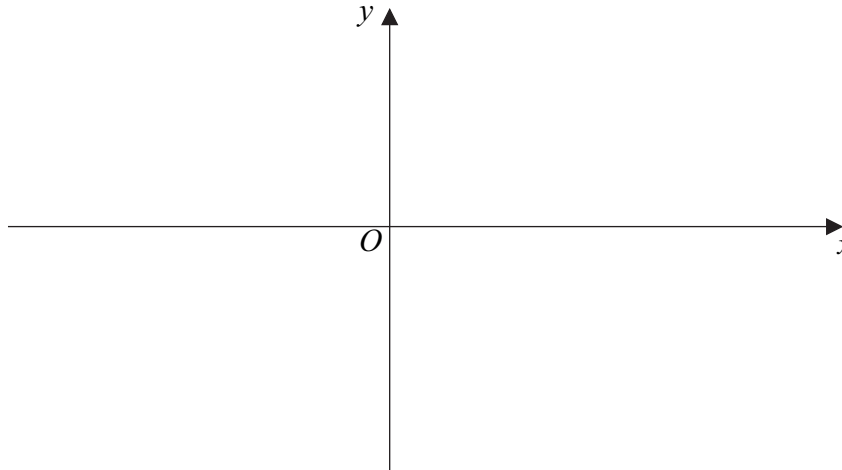


- (a) On the axes below, sketch the curve with equation $y = |f(x)|$. (2 marks)
- (b) Describe a sequence of two geometrical transformations that maps the graph of $y = f(x)$ onto the graph of $y = f(2x - 1)$. (4 marks)

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5 The function f is defined by

$$f(x) = \frac{x^2 - 4}{3}, \text{ for real values of } x, \text{ where } x \leq 0$$

(a) State the range of f . (2 marks)

(b) The inverse of f is f^{-1} .

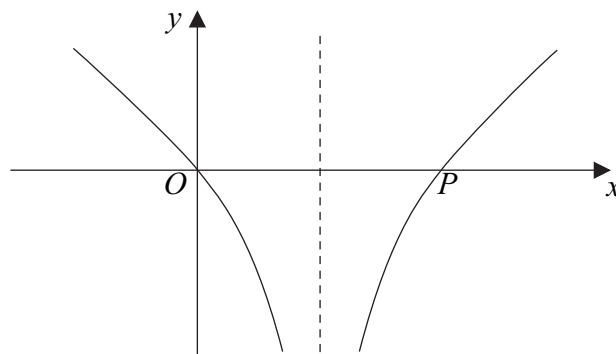
(i) Write down the domain of f^{-1} . (1 mark)

(ii) Find an expression for $f^{-1}(x)$. (3 marks)

(c) The function g is defined by

$$g(x) = \ln |3x - 1|, \text{ for real values of } x, \text{ where } x \neq \frac{1}{3}$$

The curve with equation $y = g(x)$ is sketched below.



(i) The curve $y = g(x)$ intersects the x -axis at the origin and at the point P .

Find the x -coordinate of P . (2 marks)

(ii) State whether the function g has an inverse. Give a reason for your answer. (1 mark)

(iii) Show that $gf(x) = \ln |x^2 - k|$, stating the value of the constant k . (2 marks)

(iv) Solve the equation $gf(x) = 0$. (4 marks)



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6 (a) Show that

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)}$$

can be written as $\operatorname{cosec}^2 x$.

(3 marks)

(b) Hence solve the equation

$$\frac{\sec^2 x}{(\sec x + 1)(\sec x - 1)} = \operatorname{cosec} x + 3$$

giving the values of x to the nearest degree in the interval $-180^\circ < x < 180^\circ$.

(6 marks)

(c) Hence solve the equation

$$\frac{\sec^2(2\theta - 60^\circ)}{(\sec(2\theta - 60^\circ) + 1)(\sec(2\theta - 60^\circ) - 1)} = \operatorname{cosec}(2\theta - 60^\circ) + 3$$

giving the values of θ to the nearest degree in the interval $0^\circ < \theta < 90^\circ$. (2 marks)

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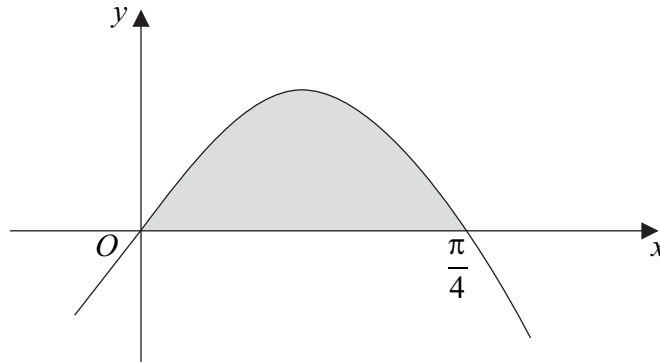


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7 A curve has equation $y = 4x \cos 2x$.

(a) Find an exact equation of the tangent to the curve at the point on the curve where $x = \frac{\pi}{4}$. (5 marks)

(b) The region shaded on the diagram below is bounded by the curve $y = 4x \cos 2x$ and the x -axis from $x = 0$ to $x = \frac{\pi}{4}$.



By using integration by parts, find the exact value of the area of the shaded region. (5 marks)

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8 (a) Show that

$$\int_0^{\ln 2} e^{1-2x} dx = \frac{3}{8}e \quad (4 \text{ marks})$$

(b) Use the substitution $u = \tan x$ to find the exact value of

$$\int_0^{\frac{\pi}{4}} \sec^4 x \sqrt{\tan x} dx \quad (8 \text{ marks})$$

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END OF QUESTIONS

