

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	



General Certificate of Education  
Advanced Level Examination  
January 2012

# Mathematics

# MPC3

## Unit Pure Core 3

Friday 20 January 2012 1.30 pm to 3.00 pm

**For this paper you must have:**

- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Time allowed**

- 1 hour 30 minutes

- Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
  - Fill in the boxes at the top of this page.
  - Answer **all** questions.
  - Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
  - You must answer the questions in the spaces provided. Do not write outside the box around each page.
  - Show all necessary working; otherwise marks for method may be lost.
  - Do all rough work in this book. Cross through any work that you do not want to be marked.

- Information**
- The marks for questions are shown in brackets.
  - The maximum mark for this paper is 75.

- Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
  - You do not necessarily need to use all the space provided.



Answer **all** questions in the spaces provided.

**1 (a)** Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for  $\int_0^3 4^x dx$ .  
(4 marks)

**(b)** A curve is defined by the equation  $y = 4^x$ . The curve intersects the line  $y = 8 - 2x$  at a single point where  $x = \alpha$ .

**(i)** Show that  $\alpha$  lies between 1.2 and 1.3. (2 marks)

**(ii)** The equation  $4^x = 8 - 2x$  can be rearranged into the form  $x = \frac{\ln(8 - 2x)}{\ln 4}$ .

Use the iterative formula  $x_{n+1} = \frac{\ln(8 - 2x_n)}{\ln 4}$  with  $x_1 = 1.2$  to find the values of  $x_2$  and  $x_3$ , giving your answers to three decimal places. (2 marks)

QUESTION  
PART  
REFERENCE



QUESTION  
PART  
REFERENCE

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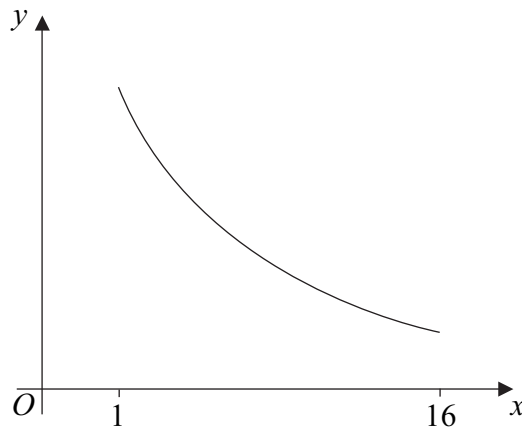
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- 2 The curve with equation  $y = \frac{63}{4x-1}$  is sketched below for  $1 \leq x \leq 16$ .



The function  $f$  is defined by  $f(x) = \frac{63}{4x-1}$  for  $1 \leq x \leq 16$ .

- (a) Find the range of  $f$ . (2 marks)
- (b) The inverse of  $f$  is  $f^{-1}$ .
- (i) Find  $f^{-1}(x)$ . (3 marks)
- (ii) Solve the equation  $f^{-1}(x) = 1$ . (2 marks)
- (c) The function  $g$  is defined by  $g(x) = x^2$  for  $-4 \leq x \leq -1$ .
- (i) Write down an expression for  $fg(x)$ . (1 mark)
- (ii) Solve the equation  $fg(x) = 1$ . (3 marks)

QUESTION  
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QUESTION  
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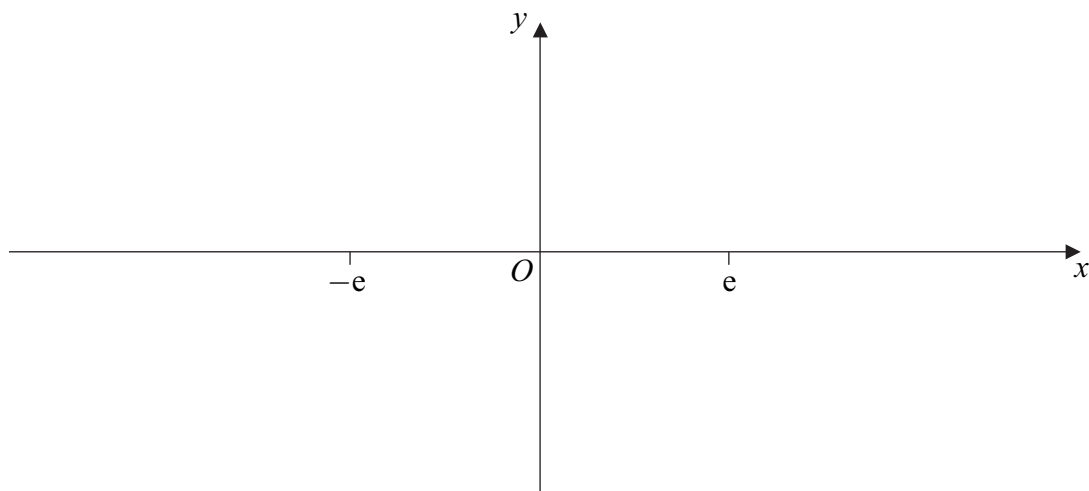








- 5 (a)** Describe a sequence of two geometrical transformations that maps the graph of  $y = \ln x$  onto the graph of  $y = 4 \ln(x - e)$ . (4 marks)
- (b)** Sketch, on the axes given below, the graph of  $y = |4 \ln(x - e)|$ , indicating the exact value of the  $x$ -coordinate where the curve meets the  $x$ -axis. (3 marks)
- (c) (i)** Solve the equation  $|4 \ln(x - e)| = 4$ . (3 marks)
- (ii)** Hence, or otherwise, solve the inequality  $|4 \ln(x - e)| \geq 4$ . (3 marks)

 QUESTION  
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**(b)**



6 (a) Given that  $x = \frac{1}{\sin \theta}$ , use the quotient rule to show that  $\frac{dx}{d\theta} = -\operatorname{cosec} \theta \cot \theta$ . (3 marks)

(b) Use the substitution  $x = \operatorname{cosec} \theta$  to find  $\int_{\sqrt{2}}^2 \frac{1}{x^2 \sqrt{x^2 - 1}} dx$ , giving your answer to three significant figures. (9 marks)

QUESTION  
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**7 (a)** A curve has equation  $y = x^2e^{-\frac{x}{4}}$ .

Show that the curve has exactly two stationary points and find the exact values of their coordinates. (7 marks)

**(b) (i)** Use integration by parts twice to find the exact value of  $\int_0^4 x^2e^{-\frac{x}{4}} dx$ . (7 marks)

**(ii)** The region bounded by the curve  $y = 3xe^{-\frac{x}{8}}$ , the  $x$ -axis from 0 to 4 and the line  $x = 4$  is rotated through  $360^\circ$  about the  $x$ -axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated. (2 marks)

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