



General Certificate of Education
Advanced Level Examination
January 2011

Mathematics

MPC3

Unit Pure Core 3

Wednesday 19 January 2011 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 (a) Find $\frac{dy}{dx}$ when $y = (x^3 - 1)^6$. (2 marks)

(b) A curve has equation $y = x \ln x$.

(i) Find $\frac{dy}{dx}$. (2 marks)

(ii) Find an equation of the tangent to the curve $y = x \ln x$ at the point on the curve where $x = e$. (3 marks)

2 A curve is defined by the equation $y = (x^2 - 4) \ln(x + 2)$ for $x \geq 3$.

The curve intersects the line $y = 15$ at a single point, where $x = \alpha$.

(a) Show that α lies between 3.5 and 3.6. (2 marks)

(b) Show that the equation $(x^2 - 4) \ln(x + 2) = 15$ can be arranged into the form

$$x = \pm \sqrt{4 + \frac{15}{\ln(x + 2)}} \quad (2 \text{ marks})$$

(c) Use the iteration

$$x_{n+1} = \sqrt{4 + \frac{15}{\ln(x_n + 2)}}$$

with $x_1 = 3.5$ to find the values of x_2 and x_3 , giving your answers to three decimal places. (2 marks)

3 (a) Given that $x = \tan(3y + 1)$:

(i) find $\frac{dx}{dy}$ in terms of y ; (2 marks)

(ii) find the value of $\frac{dy}{dx}$ when $y = -\frac{1}{3}$. (2 marks)

(b) Sketch the graph of $y = \tan^{-1} x$. (2 marks)

4 The functions f and g are defined with their respective domains by

$$f(x) = 3 \cos \frac{1}{2}x, \quad \text{for } 0 \leq x \leq 2\pi$$

$$g(x) = |x|, \quad \text{for all real values of } x$$

- (a) Find the range of f . (2 marks)
- (b) The inverse of f is f^{-1} .
- (i) Find $f^{-1}(x)$. (3 marks)
- (ii) Solve the equation $f^{-1}(x) = 1$, giving your answer in an exact form. (2 marks)
- (c) (i) Write down an expression for $gf(x)$. (1 mark)
- (ii) Sketch the graph of $y = gf(x)$ for $0 \leq x \leq 2\pi$. (3 marks)
- (d) Describe a sequence of two geometrical transformations that maps the graph of $y = \cos x$ onto the graph of $y = 3 \cos \frac{1}{2}x$. (3 marks)
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- 5 (a) Find $\int \frac{1}{3+2x} dx$. (2 marks)
- (b) By using integration by parts, find $\int x \sin \frac{x}{2} dx$. (4 marks)
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- 6 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{0.4} \cos \sqrt{3x+1} dx$, giving your answer to three significant figures. (4 marks)
- (b) Use the substitution $u = 3x + 1$ to find the exact value of $\int_0^1 x\sqrt{3x+1} dx$. (6 marks)
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Turn over ►

7 (a) Solve the equation $\sec x = -5$, giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. (3 marks)

(b) Show that the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

can be written in the form

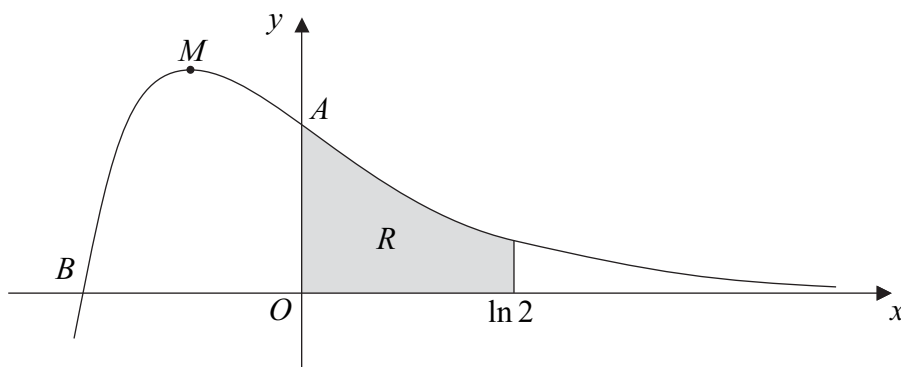
$$\sec^2 x = 25 \quad (4 \text{ marks})$$

(c) Hence, or otherwise, solve the equation

$$\frac{\operatorname{cosec} x}{1 + \operatorname{cosec} x} - \frac{\operatorname{cosec} x}{1 - \operatorname{cosec} x} = 50$$

giving all values of x in radians to two decimal places in the interval $0 < x < 2\pi$. (3 marks)

- 8 (a)** Given that $e^{-2x} = 4$, find the exact value of x . (2 marks)
- (b)** The diagram shows the curve $y = 4e^{-2x} - e^{-4x}$.



The curve crosses the y -axis at the point A , the x -axis at the point B , and has a stationary point at M .

- (i)** State the y -coordinate of A . (1 mark)
- (ii)** Find the x -coordinate of B , giving your answer in an exact form. (3 marks)
- (iii)** Find the x -coordinate of the stationary point, M , giving your answer in an exact form. (3 marks)
- (iv)** The shaded region R is bounded by the curve $y = 4e^{-2x} - e^{-4x}$, the lines $x = 0$ and $x = \ln 2$ and the x -axis.

Find the volume of the solid generated when the region R is rotated through 360° about the x -axis, giving your answer in the form $\frac{p}{q}\pi$, where p and q are integers.

(7 marks)